Regularity theory for elliptic and parabolic systems and problems in continuum mechanics

Workshop, 27th April – 30th April 2016, Telč

SCHEDULE

Thursday, April 28

Time	Speaker	Title
8:00		Breakfast
8:50		Opening
9:00	Gregory Seregin	On global weak solutions to the Cauchy problem for the Navier-Stokes equations with large L^3-initial data
10:00	Lisa Beck	Some regularity results for elliptic diagonal systems via blow-up
10:30		Coffee & Refreshment
11:00	Giuseppe Mingione	Regularity for double phase functionals
12:00	Sebastian Schwarzacher	A Gehring type result for the porous medium equation
12:30		Lunch
15:15		Coffee & Refreshment
15:45	Lars Diening	Nonlinear Calderón-Zygmund theory
16:45	Guido De Philippis	On the structure of $\mathcal{A}$-free measures and applications
17:15		Break
17:30	Akif Ibragimov	On qualitative properties of the problem in porous media, and applications
17:50	Erika Maringová	Variational problems with linear growth-regularity up to the boundary in nonconvex domains
18:10	Eero Ruosteenoja	$C^{1,\alpha}$ regularity for the Poisson problem of the normalized p-Laplacian
18:30	Peter Hästö	Harnack's inequality in generalized Orlicz spaces
19:00		Dinner

Friday, April 29

Time	Speaker	Title
8:00		Breakfast
9:00	Miroslav Bulíček	Limiting strain models in elasticity theory and variational integrals with linear growth
10:00	Vesa Julin	Harnack inequality for semilinear elliptic equations
10:30		Coffee & Refreshment
11:00	Jens Frehse	Bellman systems with mean field dependence
12:00	Thomas Schmidt	BV solutions to variational problems with obstacles or measure data
12:30		Lunch
15:15		Coffee & Refreshment
15:45	Tuomo Kuusi	The additive structure of elliptic homogenization
16:45	Jan Burczak	Existence and optimal regularity of solutions to a quasilinear Stokes system with a very weak forcing
17:05		Break
17:30	Chris van der Heide	Partial regularity for nonlinear elliptic systems with $p(x)$-growth
17:50	Šárka Nečasová	Derivation of a Navier-Stokes- Poisson system for an accretion disk
18:10	Shah Nirav	Regularity in the critical dimension
18:30	Jehan Oh	Global Gradient Estimates for Double Phase Problems in Nonsmooth Domains
19:00		Dinner

Saturday, April 30

Time	Speaker	Title
8:00		Breakfast
9:00	Andrea Cianchi	Korn and related inequalities in Orlicz spaces
10:00	Youchan Kim	Riesz potential type estimates for parabolic equations with measurable nonlinearities
10:20		Coffee & Refreshment
11:00	Alexander Ukhlov	On the First Eigenvalues of the Neumann-Laplacian in Conformal Regular Domains
11:20	Šimon Axmann	Strong solutions to the stationary Navier-Stokes equations for dense compressible fluid
11:40	Jihoon Ok	Regularity for the parabolic $p(t)$-Laplace equation
12:00		Closing
12:30		Lunch
13:30		Departure

Abstracts of main talks

Miroslav Bulíček, Limiting strain models in elasticity theory and variational integrals with linear growth. We investigate the properties of certain elliptic systems leading, a priori, to solutions that belong to the space of Radon measures. We show that if the problem is equipped with a so-called asymptotic Uhlenbeck structure, then the solution can in fact be understood as a standard weak solution, with one proviso: analogously as in the case of minimal surface equations, the attainment of the boundary value is penalized by a measure supported on (a subset of) the boundary, which, for the class of problems under consideration here, is the part of the boundary where a Neumann boundary condition is imposed.
The originally planed lecture of Ireneo Peral was replaced by this lecture.

Andrea Cianchi, Korn and related inequalities in Orlicz spaces. A standard form of the Korn inequality amounts to an estimate for the L^p norm $(1 < p < \infty)$ of the full gradient of a vector-valued function in terms of the same norm of just its symmetric part. It is well known that a result of this kind may fail if the L^p norm is replaced by a more general Orlicz norm L^A associated with a Young function A. We shall show that a Korn type inequality in Orlicz spaces can be restored if possibly different norms L^A and L^B are allowed on the two sides of the inequality, provided that the Young functions A and B satisfy suitable, necessary and sufficient balance conditions. Related inequalities for trace-free symmetric gradients, for the Bogovskii operator, and for negative Orlicz-Sobolev norms will also be discussed. Applications to strongly nonlinear systems in fluid mechanics will be outlined. Part of this talk is based on collaborations with D. Breit and L. Diening.

Lars Diening, Nonlinear Calderón-Zygmund theory. For many linear partial differential equations there is a correlation between the data and the solution in terms of a singular integral operator. Due to this fact the data and the solution are in the same function space. For example the standard Laplace equation with Lp data has solutions with second gradients in Lp. For non-linear partial differential equations this approach is not possible. However, starting with the result of Iwaniec, many similar quantitative statements have been derived for the p-Laplace equation/system. In this talk I present a novel pointwise estimate of the gradients of the solution in terms of maximal functions. Many known estimates as well as new endpoint estimates follow from this. This is a joint work with Dominic Breit, Andrea Chiani, Tuomo Kuusi, Sebastian Schwarzacher.

Jens Frehse, Bellman systems with mean field dependence. Stochastic differential games can be formulated as Nash point problems of certain, purely analytically, defined variational integrals. These integrals are defined by the so called Vlasov-McKean functionals which are defined as mean-value of the pay-offs, with the so called mean-field variable as weight. Recently, some interest came up to study pay-offs which are mean-field dependent which yields to mean-field-dependent Bellman systems. This can be used for optimization problems where the risk is taken into account.

We present a result for long time existence of such generalized Bellman systems; the difficulty is to be able to treat higher growth behaviour in the mean-field-arguments of the pay-offs. A lot of interesting analytical questions appear due to the above variational aspect, for example the application of theorems about lower semi continuity for long time existence of parabolic equations; furthermore the problem of uniqueness is not yet solved.

Tuomo Kuusi, The additive structure of elliptic homogenization. One of the principal difficulties in stochastic homogenization is transferring quantitative ergodic information from the coefficients to the solutions, since the latter are nonlocal functions of the former. In this talk, I will address this problem in a new way, in the context of linear elliptic equations in divergence form, by showing that certain quantities associated to the energy density of solutions are essentially additive. As a result, we are able to prove quantitative estimates on the first-order correctors which are optimal in both scaling and stochastic integrability. The proof of the additivity is a bootstrap argument: using the regularity theory recently developed for stochastic homogenization, we accelerate the weak convergence of the energy density, flux and gradient of the solutions as we pass to larger and larger length scales, until it saturates at the CLT scaling. This is a joint work with S. Armstrong and J.-C. Mourrat.

Giuseppe Mingione, Regularity for double phase functionals. Those mentioned in the title are integral functionals of the Calculus of Variations with the property of changing their ellipticity rate according to a variable coefficient. I will present a series of regularity results aimed at drawing a rather complete theory that parallels the one available for the p-Laplacean operator.

Gregory Seregin, On global weak solutions to the Cauchy problem for the Navier-Stokes equations with large L^3-initial data. The aim of the talk is to discuss different definitions of solutions to the Cauchy problem for the Navier-Stokes equations with the initial data belonging to the Lebesgue space $L^3(\mathbb{R}^3)$. This is a joint work with V. Šverák.

Abstracts of invited short talks

Lisa Beck, Some regularity results for elliptic diagonal systems via blow-up. We address regularity properties of (vector-valued) weak solutions to quasilinear diagonal elliptic systems, for the special situation that the inhomogeneity is allowed to be of quadratic (critical) growth in the gradient variable of the unknown. It is well-known that such systems may admit discontinuous and even unbounded solutions, when no additional structural assumption on the inhomogeneity is imposed. We here work under the condition of sum coerciveness, which has in particular some relevance in stochastic game theory. Via blow-up techniques we establish Liouville-type properties and the existence of a regular weak solution, and we finally comment on some progress for a parabolic analogue. All results presented in this talk are based on joint projects with Jens Frehse (Bonn) and Miroslav Bulíček (Prague).

Guido De Philippis, On the structure of $\mathcal{A}$-free measures and applications. I will show a general structure theorem for the singular part of $\mathcal{A}$-free Radon measures, where $\mathcal{A}$ is a linear PDE operator. By applying the theorem to suitably chosen differential operators $\mathcal{A}$, one can obtain a simple proof of Alberti's rank-one theorem and its extensions to functions of bounded deformation (BD). I will also show some consequences concerning the sharpness of Rademacher Theorem and the structure of Ambrosio–Kirchheim top-dimensional metric current in $\mathbb{R}^d$.

Vesa Julin, Harnack inequality for semilinear elliptic equations. I will discuss about Harnack type estimate for solutions of certain nonlinear equations. My main result is a natural generalization of the classical linear Harnack inequality and it gives e.g. the sharp quantification of the maximum principle. The novelty is to include the nonhomogeneous scaling of the equation into the Harnack estimate itself.

Thomas Schmidt, BV solutions to variational problems with obstacles or measure data. The talk is concerned with minimization problems for the total variation and the area functional, in which either an obstacle constraint is imposed or a lower-order term with a measure datum is present. These problems are naturally set in the space BV of functions of bounded variation, and the focus is on existence of BV minimizers, convex duality, and connections with BV supersolutions to nonlinear PDE. A crucial technical tool is a new Anzellotti type pairing between divergence-measure fields and gradient measures.

Most of these results have been obtained in collaboration with Christoph Scheven (Duisburg-Essen).

Sebastian Schwarzacher, A Gehring type result for the porous medium equation. We show that the gradient of certain super-solutions to degenerate parabolic equations of porous medium-type satisfies a reverse Hölder inequality in suitable intrinsic cylinders. Using a modification of Gehring's lemma by introducing an intrinsic Calderón-Zygmund covering argument, we are able to proof local higher integrability of the gradient.

Abstracts of short talks

Youchan Kim, Riesz potential type estimates for parabolic equations with measurable nonlinearities.

Šimon Axmann, Strong solutions to the stationary Navier-Stokes equations for dense compressible fluid. We study the existence of strong solutions to the stationary version of the Navier-Stokes system for compressible fluids with a density dependent viscosity under the additional assumption that the fluid is sufficiently dense.

Jan Burczak, Existence and optimal regularity of solutions to a quasilinear Stokes system with a very weak forcing. In contrast to linear systems, providing an L^q Calderón-Zygmund-type theory in quasilinear case is difficult for two main reasons: For 'high' q's, due to lack of smoothing property of a homogenous problem in a general case. For 'low' q's - due to lack of existence theory below the duality exponent. Focusing on the latter case, this problem has been recently resolved by Bulíček, Diening and Schwarzacher for a quasilinear, quadratic system that can be seen as an intermediary step between the Laplacian and the p-Laplacian. In my talk I will present an analogous theory for the following stationary quasilinear Stokes system

$$\begin{aligned} -\mathrm{div}\ (A(x, E(u))) + \nabla p &= -\mathrm{div}\ f \quad &&\text{in } \Omega, \\ u &= 0 &&\text{on } \partial\Omega. \end{aligned}$$

This is a joint work with M. Bulíček and S. Schwarzacher.

Akif Ibragimov, On qualitative properties of the problem in porous media, and applications. Study is dedicated to qualitative properties of the solution of the degenerate non-linear equations arising from the problems of the filtration of the fluids in porous media. Several application of the developed framework in reservoir engineering will be presented.

Erika Maringová, Variational problems with linear growth-regularity up to the boundary in nonconvex domains. The classical example of the variational problem with the linear growth is the minimal surface problem. It is well known that for smooth data such problem posses a regular (up to the boundary) solution if the domain is convex (or has positive mean curvature). On the other hand for non-convex domains, we know that there always exist data for which the solution does exists only in the space BV (the desired trace is not attained). Recently, in continuum mechanics, there were identified problems (limiting strain) that can be under certain circumstances rewritten as the variational problems with the linear growth but possibly having different structure than the minimal surface problem. We sharply identify the class of functionals for which we always have regular (up to the boundary) solution in any dimension for arbitrary $\mathcal{C}^{1,1}$ domain. Furthermore, we show that the class is sharp, i.e., whenever the functional does not belong to the class then we ca find data for which the solution does not exist.

Peter Hästö, Harnack's inequality in generalized Orlicz spaces. In this talk I present a recent result on the Harnack inequality in generalized Orlicz spaces. Special cases of the result include variable exponent growth equations and the double phase functional of Baroni, Colombo and Mingione. This is joint work with P. Harjulehto and O. Toivanen.

Šárka Nečasová, Derivation of a Navier-Stokes- Poisson system for an accretion disk. We consider the 3-D compressible barotropic Navier-Stokes-Poisson system describing the motion of a compressible rotating viscous fluid with renormalized gravitation,confined to a straight layer. The aim of this paper is to show that the weak solutions in the 3D domain converge to the strong solution of a rotating 2-D Navier-Stokes-Poisson system for all times less than the maximal life time of the strong solution of the 2-D system. It is a joint work with A. Novotný and B. Ducomet.

Shah Nirav, Regularity in the critical dimension. In this talk, we will consider bounded weak solutions ($u \in W^{1,2} \cap L^\infty(\Omega; \mathbb{R}^N)$) of the following vector-valued Euler-Lagrange system:

$$- \operatorname{div} \left(A(x,u)Du \right) = g(x,u,Du) \tag{EL}$$

in Ω a bounded open domain in $\mathbb{R}^2$. Under mild assumptions on the principal part, $A(x,u)Du$, and the inhomogeneity g, we will show that every bounded weak solution of (EL) is Hölder continuous. As the dimension of Ω is 2 and $u \in W^{1,2}(\Omega; \mathbb{R}^N)$, we are in the critical case, and hence, we cannot use the Sobolev embedding theorem to deduce regularity. This result partially resolves the open problem of whether all bounded weak solutions of non-diagonal systems, under a smallness condition on the inhomogeneity, are H"older continuous in the critical case.

Jihoon Ok, Regularity for the parabolic $p(t)$-Laplace equation. We prove the Hölder continuity of the spatial gradient of weak solutions to equations of the parabolic $p(t)$-Laplace type, where t is the time variable. We only assume that the variable exponent function $p(t)$ is log-Hölder continuous. Moreover, we directly prove it without using any perturbation argument.

Jehan Oh, Global Gradient Estimates for Double Phase Problems in Nonsmooth Domains. In this talk we consider a non-linear and non-uniformly elliptic problem in divergence form on a bounded nonsmooth domain. The problem under consideration is characterized by the fact that its ellipticity rate and growth radically change with the position. In particular, we provide a global Calderón-Zygmund type estimate for a double phase problem with (p,q)-growth. And then we discuss a similar result for the borderline case of the above problem in the case that the associated nonlinearity has a small BMO and the boundary of the domain is sufficiently flat in the Reifenberg sense.

Eero Ruosteenoja, $C^{1,\alpha}$ regularity for the Poisson problem of the normalized p-Laplacian. We prove $C^{1,\alpha}$ regularity with nearly optimal α for viscosity solutions of the inhomogeneous normalized p-Laplace equation

$$-|Du|^{2-p}\Delta_p u = f.$$

In the case $f \in L^\infty \cap C$ and $p > 1$ we use methods both from viscosity and weak theory, whereas in the case $f \in L^q \cap C$, $q > \max(n, \frac{p}{2}, 2)$, and $p > 2$ we rely on the tools from nonlinear potential theory. This is a joint work with Amal Attouchi and Mikko Parviainen.

Chris van der Heide, Partial regularity for nonlinear elliptic systems with $p(x)$-growth. Recent models in image processing, nonlinear elasticity, and fluid mechanics of highly anisotropic or otherwise inhomogeneous media require their nonlinear nature to vary from point to point. In this talk I will discuss some methods used in proving partial regularity results for nonlinear elliptic systems subject to $p(x)$-growth conditions.

Alexander Ukhlov, On the First Eigenvalues of the Neumann-Laplacian in Conformal Regular Domains. The classical results by L. E. Payne and H. F. Weinberger (1960) give the lower estimates of the first non-trivial eigenvalue of the Neumann Laplacian in convex domains in terms of its diameters. We obtain the lower estimates for the first non-trivial eigenvalue of this operator in a large class of (non)-convex domains (conformal regular domains) in terms of the hyperbolic radii. This class includes convex domains, John domains etc... We propose a new method for the estimates which is based on weighted Poincaré-Sobolev inequalities, obtained by the authors recently. (Joint work with Vladimir Gol'dshtein)