Let A be a Banach algebra

(a) A unital $\Rightarrow$ $\Delta(A)$ is a weak*-compact subset of S_{A^*}.

⌈ By Prop. 21 we have

- $\Delta(A) \subset S_{A^*}$

- $\Delta(A) = \{ h \in A^*;\ \forall x, y \in A : h(xy) = h(x) \cdot h(y)\ \&\ h(e) = 1 \}$,

so $\Delta(A)$ is weak*-closed.

So, by Banach-Alaoglu B_{A^*} is w^*-compact, hence $\Delta(A)$ is w^*-compact. ⌋

(b) $\Delta(A^+) = \{ \tilde{h};\ h \in \Delta(A) \} \cup \{ h_\infty \}$, where

$\tilde{h}(x, \lambda) = h(x) + \lambda \quad , (x, \lambda) \in A^+$

$h_\infty(x, \lambda) = \lambda \quad , (x, \lambda) \in A^+$

⌈ "$\subset$": By Prop. 21 $\tilde{h} \in \Delta(A^+)$ whenever $h \in \Delta(A)$

Moreover, clearly $h_\infty \in \Delta(A^+)$

"$\supset$" Let $g \in \Delta(A^+)$. Define $h(x) = g(x, 0)$, $x \in A$

Then $h \in A^*$, h is multiplicative

$\Rightarrow$ either $h \in \Delta(A)$ (then $g = \tilde{h}$)

or $h = 0$ (then $g = h_\infty$). ⌋

(c) A has no unit $\Rightarrow$ $\Delta(A) \subset B_{A^*}$, $\Delta(A) \cup \{0\}$ is w^*-compact

⌈ $\Delta(A) \subset B_{A^*}$ by Prop. 21

$\Delta(A) \cup \{0\} = \{ h \in A^*;\ \forall x, y \in A\ \ h(xy) = h(x) h(y) \}$

w^* closed, so w^*-compact ⌋