Functional analysis 1 – introductory information

What is this course about and what is it good for

- As obvious from the title, this course is devoted to functional analysis. It is a wide area of mathematics, investigating among others infinite-dimensional vector spaces with an additional topological structure and continuous linear mappings.
- The content of the course is formed by three advanced areas of functional analysis:
    - Topological vector spaces and weak topologies – a generalization of normed spaces. Weak topologies are important e.g. for a deeper understanding of some properties of Banach spaces. Topological vector spaces provide a theory necessary to the study of some function spaces which are not normable (algebras of continuous, smooth or holomorphic functions, Schwartz space, quasinormed spaces etc.).
    - Elements of vector integration – a generalization of the Lebesgue integral for functions with values in a Banach space. It is useful e.g. in the investigation of some partial differential equations.
    - Banach algebras and spectral theory – a wide area where analysis and algebra intersect. It is used for example to a detailed analysis of certain operators (including integral and differential ones), it is related to the group theory, Fourier analysis etc.

Assumed knowledge

It is an advanced course of a Master program, to understand it one needs a nontrivial initial knowledge. Among others:

- Elements of functional analysis – normed linear spaces, Banach and Hilbert spaces, dual spaces, bounded linear operators, basic theorems of functional analysis. This knowledge is used throughout the course.
- Elements of general topology – topological spaces, base of a topology, neighborhood base, continuous mappings, basic topological constructions, compact spaces. This is necessary to understand the first area and is used also in the third one.
- Measure theory and Lebesgue integral – abstract measure, abstract Lebesgue integral. This is a key point for the second area, but is used in the remaining two as well.
- Elements of complex analysis – holomorphic functions and their properties, Cauchy theorem and Cauchy formula etc. This knowledge serves as an important tool in the third area.