Věta 41

(a) $\Lambda \in \mathcal{S}' \Rightarrow \hat{\Lambda} \in \mathcal{S}'$

$\lceil \varphi_n \to \varphi$ v $\mathcal{S} \Rightarrow \hat{\varphi}_n \to \hat{\varphi}$ v $\mathcal{S}$ (V31(b))

$\Rightarrow \Lambda(\hat{\varphi}_n) \to \Lambda(\hat{\varphi}) \Rightarrow \hat{\Lambda}(\varphi_n) \to \hat{\Lambda}(\varphi) \rfloor$

$$\hat{\hat{\Lambda}}(\varphi) = \Lambda(\hat{\hat{\varphi}}) = \Lambda(\check{\varphi}) = \check{\Lambda}(\varphi)$$

$$\hat{\hat{\hat{\hat{\Lambda}}}}(\varphi) = \Lambda(\hat{\hat{\hat{\hat{\varphi}}}}) = \Lambda(\varphi)$$

$\Rightarrow \Lambda \mapsto \hat{\Lambda}$ je bijekce $\quad \Lambda = \widehat{(\hat{\hat{\hat{\Lambda}}})}$

linearita je jasná $\rfloor$

(b) $\Lambda_n \to \Lambda$ v $\mathcal{S}' \Leftrightarrow \hat{\Lambda}_n \to \hat{\Lambda}$ v $\mathcal{S}'$

$\lceil$ díky (a) stačí $\Rightarrow$

a to je jasné: $\hat{\Lambda}_n(\varphi) = \Lambda_n(\hat{\varphi}) \to \Lambda(\hat{\varphi}) = \hat{\Lambda}(\varphi) \rfloor$

(c) $f \in L^1(\mathbb{R}^d) \Rightarrow \widehat{\Lambda_f} = \Lambda_{\hat{f}}$

$$\lceil \widehat{\Lambda_f}(\varphi) = \Lambda_f(\hat{\varphi}) = \int_{\mathbb{R}^d} f \cdot \hat{\varphi} \overset{T26(g)}{=} \int_{\mathbb{R}^d} \hat{f} \cdot \varphi = \Lambda_{\hat{f}}(\varphi) \rfloor$$

(d) $f \in L^2(\mathbb{R}^d) \Rightarrow \widehat{\Lambda_f} = \Lambda_{P(f)}$

$$\lceil \widehat{\Lambda_f}(\varphi) = \int_{\mathbb{R}^d} f \cdot \hat{\varphi} \underset{\substack{\text{Leb. věta,}\\ \text{majoranta } |f \cdot \hat{\varphi}|}}{=} \lim_{n\to\infty} \int_{\mathbb{R}^d} \chi_{B(0,n)} f \cdot \hat{\varphi} \overset{T26(g)}{=} \lim_{n\to\infty} \int_{\mathbb{R}^d} \widehat{\chi_{B(0,n)} f} \cdot \varphi =$$

$$\underset{\chi_{B(0,n)} f \in L^1 \cap L^2}{=} \lim_{n\to\infty} \int_{\mathbb{R}^d} P(\chi_{B(0,n)} f) \cdot \varphi \underset{\substack{\chi_{B(0,n)} f \to f \text{ v } L^2 \\ \Rightarrow P(\chi_{B(0,n)} f) \to P(\cancel{\chi_{B(0,n)}} f) \text{ v } L^2}}{=} \int_{\mathbb{R}^d} P(f)\varphi \rfloor$$

(e) P polynom : $P(t) = \sum_{|\alpha| \leq N} c_\alpha t^\alpha$, $t \in \mathbb{R}^d$

$$\widehat{P(D)\Lambda}(\varphi) = P(D)\Lambda(\hat{\varphi}) = \sum c_\alpha D^\alpha \Lambda(\hat{\varphi}) = \Lambda\left(\sum c_\alpha \cdot (-1)^{|\alpha|} D^\alpha \hat{\varphi}\right)$$

$$= \Lambda\left(\breve{\breve{P}}(D)\hat{\varphi}\right) \underset{\substack{\uparrow \\ V28(c)}}{=} \Lambda\left(\widehat{\breve{P}\varphi}\right) = \hat{\Lambda}(\breve{P}\varphi) = \breve{P} \cdot \hat{\Lambda}(\varphi)$$

$$\widehat{P \cdot \Lambda}(\varphi) = P\Lambda(\hat{\varphi}) = \Lambda(P \cdot \hat{\varphi}) = \Lambda\left(\widehat{\breve{\breve{\breve{P}}}(D)\varphi}\right) =$$

$$\breve{\breve{\breve{P}}} = P$$

$$= \hat{\Lambda}\left(\breve{\breve{\breve{P}}}(D)\varphi\right) = \hat{\Lambda}\left(\sum c_\alpha \cdot (-i)^{|\alpha|} D^\alpha \varphi\right) =$$

$$= \sum_{\alpha} c_\alpha (-i)^{|\alpha|} \cdot (-1)^{|\alpha|} D^\alpha \hat{\Lambda}(\varphi) = \sum c_\alpha \cdot (i)^{|\alpha|} D^\alpha \hat{\Lambda}(\varphi) = \breve{P}(D)\hat{\Lambda}(\varphi).$$