Introduction

This doctoral thesis consists of the following four research papers: *Remark on the Point of Continuity Property II* (joint paper with P.Holický, Bull. Acad. Polon. Sci., **50** (1995), 105–111), *Note on Connections of Point of Continuity Property and Kuratowski Problem on Function Having the Baire Property* (to appear in Acta Univ. Carolinae, Math. et Phys., **36**(1), 1997), *New Examples of Hereditarily t-Baire Spaces* (submitted for publication to Bull. Acad. Polon. Sci.), *Stegall Compact Spaces Which Are Not Fragmentable* (submitted for publication to Topol. Appl.), *Few remarks on structure on certain spaces of measures*(preprint). The thesis is also equipped by a short introduction.

The work contains results dealing with several continuity-like properties of mappings between topological spaces. The first three papers are concerned with maps of topological spaces into metric spaces and investigate connections between measurability and continuity, while in the last two sections sets of continuity points of upper semi-continuous compact-valued maps are studied. So let us start with precise definitions of the properties in question.

Let $f : X \to M$ be a mapping of a topological space X into a metric space M. If $\mathcal{H}$ is a collection of subsets of X, we say that f is *$\mathcal{H}$-measurable* if, whenever U is an open subset of M, we have $f^{-1}(U) \in \mathcal{H}$. Now we give a list of some collections of sets we are interested in. By $\mathcal{F}$ (resp. $\mathcal{G}$) we denote the collection of all closed (resp. open) subsets of X. The family $\mathcal{F} \wedge \mathcal{G}$ consists of the sets of the form $F \cap G$ with $F \in \mathcal{F}$ and $G \in \mathcal{G}$. The symbols $\mathcal{F}_\sigma$, $(\mathcal{F} \wedge \mathcal{G})_\sigma$ denote the collections of all countable unions of elements of $\mathcal{F}$, $\mathcal{F} \wedge \mathcal{G}$, respectively. Finally, by $(\mathcal{F} \wedge \mathcal{G})_{\sigma\text{-scattered}}$ we mean the family of all σ-scattered unions of members of $\mathcal{F} \wedge \mathcal{G}$. The $\mathcal{F}_\sigma$-measurable functions are called *functions of the Borel class one*, the $(\mathcal{F} \wedge \mathcal{G})_{\sigma\text{-scattered}}$-measurable maps are *maps of the extended Borel class one*.

A classical theorem of Baire points out connection of $\mathcal{F}_\sigma$-measurability with the point of continuity property (the mapping f is said to have the *point of continuity property (PCP)* if for every nonempty closed $F \subset X$ the restriction $f \restriction F$ has a point of continuity). We recall here a version of this theorem.

Theorem (Baire). *Let X be a complete metric space, M a separable metric space and $f : X \to M$ a mapping. Then f is $\mathcal{F}_\sigma$-measurable if and only if it has the point of continuity property.*

In [Ha] and other works it is investigated whether a theorem like this holds if one drops (or weakens) the assumptions of the complete metrizability of the domain and of the separability of the range. Since, for example, the characteristic function of any open set has PCP but the open set need not be $\mathcal{F}_\sigma$ in a nonmetrizable space, it follows that the 'if' part need not hold. R.W.Hansell in [Ha] deals with $(\mathcal{F} \wedge \mathcal{G})_\sigma$-measurable maps and gives a characterization of those spaces X for which every function having PCP with values in a metric space is $(\mathcal{F} \wedge \mathcal{G})_\sigma$-measurable. He also proved there that the inverse implication holds if X is *hereditarily Baire* (i.e. each nonempty closed subset of X is a Baire space) and either M is separable or X has countable tightness. It can be observed ([HK]) that, whenever X satisfies the 'only if' part of the Baire theorem for $(\mathcal{F} \wedge \mathcal{G})_\sigma$-measurable mappings with separable range, then X is necessarily hereditarily Baire.

In [Ho] and [Ko1] the $(\mathcal{F} \wedge \mathcal{G})_{\sigma\text{-scattered}}$-measurable functions are studied. This is really the most general notion of Borel class one since, as it is proved e.g. in [Ko1], every function with PCP is necessarily $(\mathcal{F} \wedge \mathcal{G})_{\sigma\text{-scattered}}$-measurable. Moreover, if the domain is metrizable then the $(\mathcal{F} \wedge \mathcal{G})_{\sigma\text{-scattered}}$-measurable maps coincide with $\mathcal{F}_\sigma$-measurable ones. But the main question studied in this area is when the inverse implication holds. It is not hard to improve the Hansell result to show ([Ko1]) that if X is hereditarily Baire and M is separable then every extended Borel class one function $f : X \to M$ has PCP.

Now we will briefly exhibit each paper.

Remark on the Point of Continuity Property II

The aim of this paper is to prove some positive results on generalizations of the Baire theorem. We make use of the fact that $(\mathcal{F} \wedge \mathcal{G})_{\sigma\text{-scattered}}$ sets have the *strong Baire property in the restricted sense (SBPR)* (see [Ko1], where SBPR-sets are called almost H-sets). Recall that a set A in a topological space X has SBPR if, for every $F \subset X$, one can write $A \cap F = G \cup P$ where G is relatively open and P meager in F. In

[Ko1] it is proved that on a hereditarily Baire space the $(\mathcal{F} \wedge \mathcal{G})_{\sigma\text{-scattered}}$-measurable functions coincide with SBPR-measurable maps. This enables us to give the following characterization.

Proposition ([HK]). *Let X be a topological space and κ be an infinite cardinal. Then the following two conditions are equivalent.*

(1) Every extended Borel class one map of X into a metric space of weight $\leq \kappa$ has PCP.

(2) Whenever F is nonempty closed subset of X and $\mathcal{E}$ a disjoint collection of sets meager in F, such that $\operatorname{card} \mathcal{E} \leq \kappa$ *and for every $\mathcal{E}' \subset \mathcal{E}$ the union $\bigcup \mathcal{E}'$ has SBPR (in F), then $\bigcup \mathcal{E}$ has empty interior in F.*

Using this we establish two theorems, one concerning spaces of a specified tightness, the other one hereditarily ccc spaces. Let us recall that the *tightness* of a topological space X is the smallest cardinal τ such that whenever $A \subset X$ and $x \in \overline{A}$ then there is $C \subset A$ of cardinality at most τ with $x \in \overline{C}$. Further we recall that a space X is said to be *ccc* if every disjoint family of nonempty open subsets of X is at most countable. The space X is *hereditarily ccc* if each its subspace is ccc (it is easy to see that it is sufficient to check it for closed subspaces).

Now let us give a result on spaces with a specified tightness (which is for the case of regular X due to P.Holický).

Theorem ([HK]). *Let τ be an infinite cardinal and X a topological space of tightness at most τ. If every extended Borel class one function of X into a metric space of weight $\leq \tau$ has PCP, then the same holds for maps into any metric space.*

From this immediately a slight improvement of a result from [Ha] follows.

Corollary ([HK]). *If X is a hereditarily Baire space with countable tightness, then any extended Borel class one map of X into a metric space has PCP.*

The result on hereditarily ccc spaces reads as follows.

Theorem ([HK]). *Let X be a hereditarily Baire and hereditarily ccc space. Then the following holds.*

(i) Every extended Borel class one map of X into a metric space of weight less than the least weakly inaccessible cardinal has PCP.

(ii) If the tightness of X is less than the least weakly inaccessible cardinal, then every extended Borel class one map of X into any metric space has PCP.

Let us recall that an uncountable cardinal is *weakly inaccessible* if it is regular and limit, so for example $\aleph_1$ is less than the least weakly inaccessible cardinal. Also, let us notice that it is consistent with the set theory that there are no weakly inaccessible cardinals.

The assertion (ii) demonstrates usefulness of the result on tightness, since it is an obvious consequence of (i) and the previous theorem.

Note on Connections of Point of Continuity Property and Kuratowski Problem on Function Having the Baire Property

In this paper we establish the equivalence of the generalized Baire theorem with a generalization of a theorem of Kuratowski. This one deals with functions having the Baire property. A function $f : X \to M$ is said to have *the Baire property* (or, equivalently, to be *BP-measurable*) if the inverse image of every open set in M has the Baire property in X. Similarly as Borel class one functions also the BP-measurable maps enjoy a continuity property.

Theorem (Kuratowski). *Let X be a topological space, M be a separable metric space, $f : X \to M$. Then f has the Baire property if and only if there is $P \subset X$ meager such that $f \restriction X \setminus P$ is continuous.*

In this case the 'if' part holds even for nonseparable M and the 'only if' part has been investigated extensively for example in [Fr, Section 7]. In [FK] it is proved that the negative answer (i.e., existence of a function for which the 'only if' part fails) is equiconsistent with the existence of a measurable cardinal.

Using some properties of ideal topologies we prove the following theorem.

Theorem ([Ka1]). *Let κ be an infinite cardinal. Then the following conditions are equivalent:*

(1) There is a topological space X, a metric space M of weight at most κ, and a function $f : X \to M$ having the Baire property, such that there is no meager $N \subset X$ with $f \restriction (X \setminus N)$ being continuous.

(2) There is a hereditarily Baire space X, a metric space M of weight at most κ, and an $(\mathcal{F} \wedge \mathcal{G})_{\sigma\text{-scattered}}$-measurable function $f : X \to M$ which has not the point of continuity property.

(3) There is a hereditarily Baire space X, a metric space M of weight at most κ, and an $(\mathcal{F} \wedge \mathcal{G})$-measurable function $f : X \to M$ which has not the point of continuity property.

Moreover, these conditions remain equivalent when to each one the assumption that the space X is Hausdorff is added.

In particular, using [FK], we get that the existence of an extended Borel class one function on a hereditarily Baire space for which the generalized Baire theorem fails is equiconsistent with the existence of a measurable cardinal. The method of the proof of Theorem enables us to modify several known examples ([Fr], [FK]) to get the following examples of even $\mathcal{F}_\sigma$-measurable functions without PCP.

Example ([Ka1]). *Let there exist a measurable cardinal κ. Then there is a Hausdorff hereditarily Baire space X and an $\mathcal{F}_\sigma$-measurable function of X into the discrete space of cardinality κ which has not PCP.*

Example [Ka1]. *Let "ZFC + there is a measurable cardinal" is consistent. Then so is "ZFC + (i) & (ii) & (iii)", where*

(i) there is no real-valued-measurable cardinal,

(ii) there is an $\mathcal{F}_\sigma$-measurable function of a hereditarily Baire Hausdorff space into the discrete metric space of cardinality $\aleph_1$ which has not PCP,

(iii) if X is such that every extended Borel class one function of X into a metric space of weight at most $\aleph_1$ has PCP, then the same holds for extended Borel class one maps of X into any metric space.

The second example shows that the separability of the range in the generalized Baire theorem cannot be weakened without some additional assumptions on the domain. In both examples the constructed spaces are Hausdorff but not regular. It seems to be unknown whether at least $\mathcal{F}_\sigma$-measurable functions on hereditarily Baire regular spaces have PCP. The next paper shows that it is not the case for $(\mathcal{F} \wedge \mathcal{G})_\sigma$-measurable maps.

New Examples
of Hereditarily t-Baire Spaces

The main goal of this paper is to answer a question of G.Koumoullis posed in [Ko1]. He introduced the class of hereditarily t-Baire spaces and proved that they satisfy a strong version of the Baire theorem. Firstly, let us give the definition.

Let X be a Hausdorff space. By $M_t(X)$ (resp. $P_t(X)$) we denote the space of all finite positive Radon measures (resp. Radon probability measures) on X. These spaces are considered with the *weak topology* (by some authors called *narrow topology*), i.e. the weakest topology such that the mapping $\mu \mapsto \mu(X)$ is continuous and maps $\mu \mapsto \mu(G)$ are lower semi-continuous for every open G. If X is completely regular then this topology coincides with the $w^\star$-topology induced by bounded continuous functions on X. These spaces are studied thoroughly for example in [Ko2]. Following [Ko1] we call a Hausdorff space X *t-Baire* if $M_t(X)$ (or, equivalently, $P_t(X)$) is a Baire space. In [Ko2] it is proved that X is t-Baire if and only if $M_t(X)$ (or, equivalently, $P_t(X)$) is of second category in itself. The space X is *hereditarily t-Baire* if each its nonempty closed subset is t-Baire.

G.Koumoullis proved the following generalization of the Baire theorem.

Theorem ([Ko1]). *Let X be a hereditarily t-Baire space, M a metric space of nonmeasurable cardinality, and $f : X \to M$ a function of the extended Borel class one. Then f has the point of continuity property.*

The class of hereditarily t-Baire spaces contains all compact Hausdorff spaces, all Čech complete and even 'hereditarily almost Čech complete' spaces. Recall that a Hausdorff completely regular space is *hereditarily almost Čech complete* if each its nonempty closed subset contains a dense Čech complete subspace. We introduce a new subclass of hereditarily t-Baire spaces, which need not be hereditarily almost Čech complete. To this end we define the following notions.

Let Y be a Hausdorff space and $X \subset Y$. We say that X is *countably closed* in Y if for each $C \subset X$ countable we have $\overline{C} \subset X$. We say that X is *K_σ-closed* in Y if for each $C \subset X$ which is the countable union of compacts (i.e. K_σ) we have $\overline{C} \subset X$.

Using these notions we can formulate the following result.

Proposition ([Ka2]). *If X is K_σ-closed in a compact Hausdorff space then for each closed $\emptyset \neq F \subset X$ the space $P_t(F)$ is countably closed in a compact space (hence is countably compact) and thus X is hereditarily t-Baire.*

This leads us to the following example.

Example ([Ka2]). *The space $[0, \omega_1)^A$ (we consider $[0, \omega_1)$ with the order topology) is hereditarily t-Baire for every set A, and if A is uncountable it has not the Baire property in (some) any compactification (and therefore is not almost Čech complete).*

Further we prove a stability result of a subclass of t-Baire spaces. Before stating it let us recall the definition of weakly α-favorable spaces.

A space X is called *weakly α-favorable* if there is a mapping τ which assigns to each finite sequence $G_1, \ldots, G_n$ of nonempty open subsets of X a nonempty open $\tau(G_1, \ldots, G_n) \subset G_n$ such that, whenever $G_n, n \in \mathbb{N}$, is a sequence of nonempty open subsets of X satisfying $G_{n+1} \subset \tau(G_1, \ldots, G_n)$ for every n, then $\bigcap_{n \in \mathbb{N}} G_n \neq \emptyset$. A space X is called *α-favorable* if there is a mapping σ assigning to each nonempty open G its nonempty open subset $\sigma(G)$ such that the mapping $\tau(G_1, \ldots, G_n) = \sigma(G_n)$ satisfies the above conditions. Weakly α-favorable spaces form a subclass of Baire spaces which is stable with respect to arbitrary products. In view of this we proved the following result.

Proposition ([Ka2]). *Let $(X_a \mid a \in A)$ be a collection of Hausdorff spaces such that $P_t(X_a)$ is weakly α-favorable for each $a \in A$. Put $X = \bigoplus_{a \in A} X_a$. Then $P_t(X)$ is weakly α-favorable.*

Using this and the previous proposition we get the example which answers the Koumoullis' question whether the cardinality restriction in his theorem can be dropped.

Example ([Ka2]). *Suppose that there is a measurable cardinal κ. Then there is a Hausdorff completely regular space X satisfying the following conditions:*

(i) Each nonempty closed subset of X is α-favorable.

(ii) For each nonempty closed $F \subset X$ the space $P_t(F)$ is weakly α-favorable (and hence X is hereditarily t-Baire).

(iii) There is a partition $(E_\xi)_{\xi<\kappa}$ *of* X *into closed nowhere dense sets such that* $\bigcup_{\xi\in A} E_\xi$ *is either closed nowhere dense or open dense in* X *for every* $A \subset [0,\kappa)$.

If X *satisfies the conditions (i)–(iii), if* M *is the set* $[0,\kappa)$ *endowed with the discrete metric, and* $f : X \to M$ *is the mapping assigning to each* $x \in X$ *that* ξ *for which* $x \in E_\xi$*, then* $f^{-1}(A)$ *is either open or closed for each* $A \subset M$ *and* f *has no continuity point.*

The function constructed in the previous example is $(\mathcal{F} \wedge \mathcal{G})$-measurable (even "$(\mathcal{F} \cup \mathcal{G})$-measurable") but not $\mathcal{F}_\sigma$-measurable. Also, the space X is not almost Čech complete, hence it remains open whether the generalized Baire theorem holds for hereditarily almost Čech complete spaces, or at least for compact Hausdorff spaces. While for compact spaces (and, for example, Čech complete ones) it is known that $\mathcal{F}_\sigma$-measurable functions on such spaces have PCP, the same question for $(\mathcal{F} \wedge \mathcal{G})_\sigma$-measurable maps is open.

Stegall Compact Spaces Which Are Not Fragmentable

In this work we give consistent examples of compact Hausdorff spaces which are not fragmentable but belong to the Stegall class $\mathcal{S}$. At first let us recall the definitions.

The topological space Y is *fragmented* by a metric ρ if any nonempty subset of Y contains a nonempty relatively open set of arbitrarily small (ρ-)diameter. The space Y is said *fragmentable* if it is fragmented by some metric.

The class $\mathcal{S}$ is defined via minimal usco mappings. So we recall the definition of such mappings.

Let $\varphi : X \to Y$ be a set-valued mapping acting between two Hausdorff topological spaces. We say that φ is an *usco mapping (upper semi-continuous compact-valued)* if, for every $x \in X$, $\varphi(x)$ is a nonempty compact subset of Y, and whenever $\varphi(x) \subset U$, where U is open in Y, there is a neighborhood V of x such that $\varphi(V) \subset U$ (or, equivalently, $\varphi^{-1}(F) = \{x \in X \mid \varphi(x) \cap F \neq \emptyset\}$ is closed in X whenever F is closed in Y). An usco mapping $\varphi : X \to Y$ is called *minimal* if it is minimal with respect to the inclusion (i.e. if $\psi : X \to Y$ is usco such that $\psi(x) \subset \varphi(x)$ for every $x \in X$, then $\psi = \varphi$).

The space Y belongs to the *Stegall class* $\mathcal{S}$ if, whenever X is a Baire space and $\varphi : X \to Y$ a minimal usco mapping, then φ is singlevalued at points of a residual set (or, equivalently, at least at one point).

The class $\mathcal{S}$ was introduced by C.Stegall in [S1] while studying weak Asplund spaces. The motivation in this direction is the fact that X is a weak Asplund space provided the dual unit ball endowed with the $w^\star$-topology belongs to the class $\mathcal{S}$. It is easy to see that fragmentable spaces form a subclass of $\mathcal{S}$. It was proved in [R] that, if K is a fragmentable compact space, then the dual unit ball $\left(B_{\mathcal{C}(K)^\star}, w^\star\right)$ is fragmentable, too. We get our examples by a modification of the construction of the "double arrow" space.

Let $K = [0,1]$, $A \subset (0,1)$ be arbitrary. Put

$$K_A = ((\{0\} \cup A) \times \{1\}) \cup ((0,1] \times \{0\}) .$$

We define on K_A a topology. The neighborhood basis of $(t,\varepsilon) \in K_A$ will be

(i) $\{(t-\Delta, t+\Delta) \times \{0,1\} \cap K_A \mid \Delta > 0\}$ if $t \in (0,1) \setminus A$ (and $\varepsilon = 0$);

(ii) $\{((t-\Delta, t] \times \{0\} \cup (t-\Delta, t) \times \{1\}) \cap K_A \mid \Delta > 0\}$ if $t \in A \cup \{1\}$ and $\varepsilon = 0$;

(iii) $\{((t, t+\Delta) \times \{0\} \cup [t, t+\Delta) \times \{1\}) \cap K_A \mid \Delta > 0\}$ if $t \in A \cup \{0\}$ and $\varepsilon = 1$.

Proposition ([Ka3]). *Let $K = [0,1]$ and $A \subset (0,1)$ be arbitrary. Let K_A be as above, with the topology defined by (i)–(iii). Then K_A is a first countable hereditarily Lindelöf and hereditarily separable compact Hausdorff space.*

In the framework of the spaces K_A we give the following characterizations.

Proposition ([Ka3]). *Let $K = [0,1]$ and $A \subset (0,1)$. Then the following assertions are equivalent:*

(a) A is countable;

(b) K_A is metrizable;

(c) K_A is fragmentable.

Proposition ([Ka3]). *Let $K = [0,1]$ and $A \subset (0,1)$. Then the following assertions are equivalent.*

(1) K_A belongs to the class $\mathcal{S}$.

(2) For every Baire space X and every $f : X \to A$ continuous there is $U \subset X$ nonempty open such that the set $f(U)$ has maximum or minimum.

Proposition ([Ka3]). *Let $K = [0,1]$ and $A \subset (0,1)$. Then the following conditions are equivalent.*

(i) Every closed subset of K_A contains a dense completely metrizable subspace.

(ii) A is perfectly meager.

Let us recall that a set $A \subset \mathbb{R}$ is called *perfectly meager* if every dense in itself subset of A is meager in itself, or, equivalently, if for every $P \subset \mathbb{R}$ the intersection $P \cap A$ is meager in P. This definition follows [Ku] where it is also proved that there is an uncountable perfectly meager set.

The condition (i) in the previous proposition is related to the problem via the following statement due to C.Stegall.

Proposition ([S2]). *Let K be a compact Hausdorff space. Then the following conditions are equivalent.*

(1) K is Stegall with respect to completely regular spaces (i.e. every minimal usco map from a completely regular Baire space into K is singlevalued at points of a residual set).

(2) For every complete metric space M and every closed subset $F \subset K \times M$, the set F contains a dense completely metrizable subspace.

So the mentioned condition (i) is necessary for K_A to belong to $\mathcal{S}$.

If we consider a strengthening of the condition (2), namely the condition

(*) For any Baire space X and any $f : X \to A$ continuous there is $U \subset X$ open such that f is constant on U,

we get the following.

Proposition ([Ka3]. *(a) If A is a $\mathcal{Q}$-set and* $\operatorname{card} A = \aleph_1 = \aleph_1^L$ *(or, more generally,* $\operatorname{card} A$ *is less than the least inaccessible cardinal in L) then A satisfies (*) with respect to the class of all Baire spaces.*

(b) ([NP]*) If A is a coanalytic set with no perfect subset then A satisfies (*) with respect to the class of all completely regular Baire spaces.*

Let us recall that a $\mathcal{Q}$-set is a set whose every subset is relatively $\mathcal{F}_\sigma$. Such an uncountable set exists under MA & $\neg$ CH (see [MS]). An uncountable coanalytic set without perfect subset exists under $V = L$. So we get the following theorem.

Theorem ([Ka3]). *(1) Assume Martin's axiom and the negation of continuum hypothesis, and moreover $\aleph_1 = \aleph_1^L$ (or, more generally, $\aleph_1$ is less than the least inaccessible cardinal in L). Then there is a Hausdorff compact space K which is not fragmentable but belongs to $\mathcal{S}$.*

(2) Assume $V = L$. Then there is a Hausdorff compact space K which is not fragmentable but is Stegall with respect to the class of all completely regular Baire spaces.

So we have consistent examples of Stegall compact spaces which are not fragmentable. There are two natural questions. Firstly, if there is an absolute example. A candidate is, of course, the space K_B for B uncountable and perfectly meager. However, under some set-theoretical assumptions no uncountable subset of $[0,1]$ satisfies the condition (*). The second question is whether the dual unit ball of some of the spaces K_A (with A uncountable) belongs to $\mathcal{S}$, too. In the following paper we give some partial positive results in this direction.

Few remarks on structure of certain spaces of measures

In this note we study the structure of the space of positive Radon measures on the spaces K_B. We prove also two theorems which we use to prove our main result but which may be of an independent interest. The first one is a result on Baire-property-additive families in compact spaces.

Theorem 1 ([Ka4]). *(i) Let K be a compact Hausdorff space such that $c(K) < 2^{\aleph_0}$ and $\operatorname{card} K$ is less than the least weakly inaccessible cardinal. Then, whenever $\mathcal{E}$ is a point-finite family of meager sets in K such that the union of every subfamily has the Baire property in K, the union $\bigcup \mathcal{E}$ is meager in K, too.*

(ii) Let K be a compact Hausdorff space of cardinality $\leq 2^{\aleph_0}$ and $\mathcal{E}$ a point-finite family of meager subsets of K such that $\bigcup \mathcal{E}'$ has the restricted Baire property for every $\mathcal{E}' \subset \mathcal{E}$. Then $\bigcup \mathcal{E}$ is meager.

The second theorem is concerned with descriptive properties of certain sets of measures.

Theorem 2([Ka4]). *If K is a compact Hausdorff space and $A \subset K$ is Borel (Suslin-$\mathcal{F}$, Suslin-$\mathcal{B}$, co-Suslin-$\mathcal{F}$, co-Suslin-$\mathcal{B}$) then so are the sets* $\{\mu \in \mathcal{M}_+(K) \mid (\exists a \in A)(\mu(\{a\}) > c)\}$ *and* $\{\mu \in \mathcal{M}_+(K) \mid (\exists a \in A)(\mu(\{a\}) \geq c)\}$ *for every* $c \geq 0$.

We use these two theorems and several auxiliary results to prove the main result which is the following example.

Example ([Ka4]). *Suppose* MA & $\neg$CH & $\aleph_1 = \aleph_1^L$. *Let* $B \subset (0,1)$ *have cardinality* $\aleph_1$. *Then* K_B *is Stegall, non-fragmentable and every closed ccc subset of* $\mathcal{M}_+(K_B)$ *contains a dense completely metrizable subset.*

This is a partial result in the attempt to find a set B such that $\mathcal{M}_+(K_B)$ (and hence also $\left(B_{\mathcal{C}(K_B)^\star}, w^\star\right)$) is Stegall. However, as shown also in [Ka4], there are compact subsets of $\mathcal{M}_+(K_B)$ which are "locally non-ccc".

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