

Longitudinal and Panel data | (NMST 422)

Summer Term 2024 | Department of Probability and Mathematical Statistics



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Topics to cover

- ❑ Linear regression overview and multivariate linear regression
- ❑ Longitudinal data and their representation
- ❑ Linear mixed effect models (marginal vs. hierarchical)
- ❑ GLM overview and generalized estimating equations (GEE)
- ❑ GLMM for binary and count data
- ❑ Missing data concepts
- ❑ Bayesian approaches
- ❑ Further extensions & generalizations

Bibliography

- ❑ **Diggle, P.J, Heagerty, P. Liang, K.Y., and Zeger, S. (2022)**
Analysis of Longitudinal Data. Oxford University Press
- ❑ **Fitzmaurice, G.M., Laird, N.M., and Ware, J.H. (2012)**
Applied Longitudinal Analysis. John Wiley & Sons
- ❑ **Hardin, J.W. and Hilbe, J.M. (2007)**
Generalized Linear Model and Extensions. StataPress.
- ❑ **Kulich, M. (2022)**
NMST432 Advanced Regression Models: Extended Course Notes
www.karlin.mff.cuni.cz/~kulich/vyuka/pokreg/index.html (18.02.2022)
- ❑ **Pinheiro, J. and Bates, D. (2006)**
Mixed-effects models in S and S-PLUS. Springer Science & Business

Bibliography

- ❑ **Diggle, P.J, Heagerty, P. Liang, K.Y., and Zeger, S. (2022)**
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Mixed-effects models in S and S-PLUS. Springer Science & Business
- ❑ **Additional studying material**
Details will be given when/if needed

General course information

□ General conditions

- enrollment into the corresponding SIS group
- pre-requisite: Linear regression course (NMSA 407)

□ Credit requirements

- in-person lab session attendance
- active participation
- individual project assignment (more details at the first lab session)

□ Final Course Exam

- final exams at the end of the term (course credit required)
- the exam is composed of two parts – written and oral
- written part contains theory and examples from the lectures
- oral part includes a discussion of the written part and the project solution

Lecture organization

❑ In-person teaching

- ❑ PDF slides (available apriori on the course web page)
- ❑ hand written notes (on the board in the class)
- ❑ additional literature to read/study

❑ Individual work

- ❑ some lectures (lab sessions respectively) not taking place in person
- ❑ individual assignment for studying/working given instead
- ❑ all necessary information will be given when needed

↔ The PDF slides primarily serve as an extended (detailed) syllabus for the lecture. Additional material and specific pieces of information (such as calculations and derivations) will be given by hand.

The PDF slides do not comprehend all necessary information required for the exam!

Lecture 1 | 19.02.2024

Linear regression overview (i.i.d. and/vs. correlated data)

What is the linear regression in general?

- ❑ **historically** (Francis Galton)
- ❑ **mathematically** (functional relationship)
- ❑ **geometrically** (orthogonal projection)
- ❑ **numerically** (least squares/normal equations)
- ❑ **probabilistically** (conditional expectation)
- ❑ **statistically** (estimation of the expectation)
- ❑ **computationally** (matrix QR decomposition)

Theoretical perspective:

Empirical perspective

probabilistic model \implies model interpretation
data \implies model \implies inference \implies interpretation

Linear regression model(s)

- ordinary linear regression (theoretical/generic model)

$$Y = \alpha + \beta X + \varepsilon$$

- ordinary linear regression (theoretical/probabilistic model)

$$E[Y|X] = \alpha + \beta X$$

- ordinary linear regression (empirical/statistical model)

$$Y_i = \alpha + \beta X_i + \varepsilon_i$$

↪ recall the common notation, alternative model definitions, formulations for iid errors (random sample respectively), typical assumptions, and the consequent theoretical properties of the estimates $\hat{\alpha}$ and $\hat{\beta}$ (respectively $\hat{\beta}$).

Generalization for correlated data

- In practice: **correlated observations** (e.g., multiple observations)
(paired t-test and further generalizations, repeated measures in general)

- Example: X_1, \dots, X_n (**random sample**) – estimate of the mean: \bar{X}_n
(what is the mean and the variance of the corresponding estimate?)
 - $\text{Var}\bar{X}_n$ if $\text{cor}(X_i, X_j) = 0$ (i.e., independence, random sample)
 - $\text{Var}\bar{X}_n$ if $\text{cor}(X_i, X_j) = 1$ (i.e., $\text{cov}(X_i, X_j) = \sigma^2$)
 - $\text{Var}\bar{X}_n$ if $\text{cor}(X_i, X_j) = \gamma \in (0, 1)$ (i.e., $\text{cov}(X_i, X_j) = \sigma^2$)

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 - **Now, what if $\gamma < 0$?**

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 - **Now, what if $\gamma < 0$?**

\leftrightarrow the variance of a random variable $X \in \mathbb{R}$ is supposed to be always positive...
However, for the random vector $\mathbf{X} \in \mathbb{R}^p$ the condition becomes more strict...

\Rightarrow **the variance-covariance matrix must be positive definite!**

What kind of consequences does it imply?

(*curse of dimension problem*)

Data structures beyond random samples...

❑ random sample (i.i.d. data)

- ❑ typical for many simple (but very practical) problems
- ❑ simple theory behind, straightforward proofs, easy implementation
- ❑ however, not always realistic ...

❑ correlated (i.e., dependent) data

- ❑ different forms of dependence (time/spatial)
- ❑ group dependent data (clustered/repeated/longitudinal/panel data)
- ❑ however, still i.i.d. in some (well-formlated) sense

❑ n.i.n.i.d. data

- ❑ generally not independent and not identically distributed observations
- ❑ complex and sophisticated data structures (occurring in practical situations)
- ❑ very typical for panel data with dependent panels for instance

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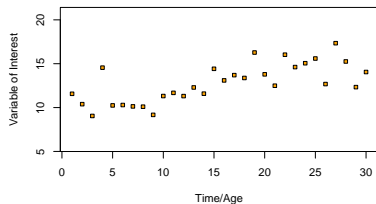
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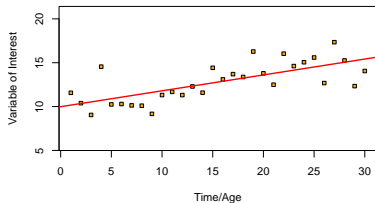
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Question: In which category would you expect the **time series** to appear?

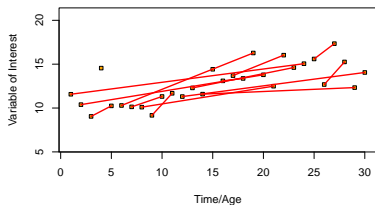
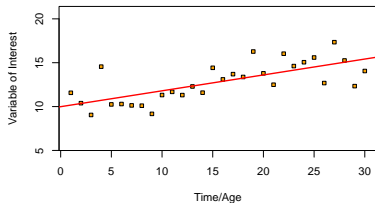
Example: independent/paired/panel data



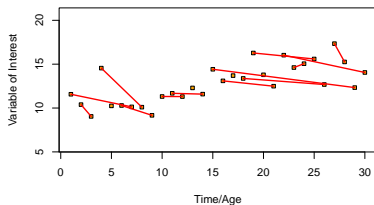
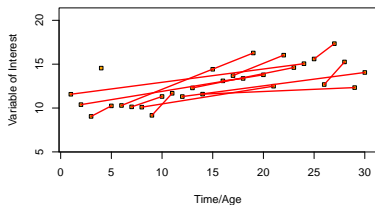
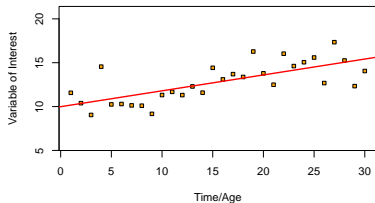
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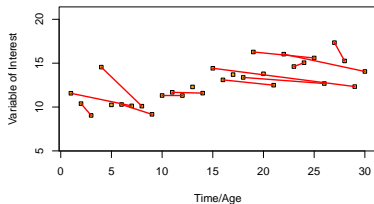
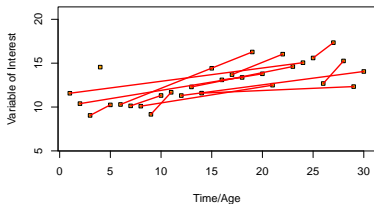
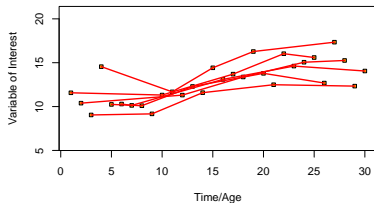
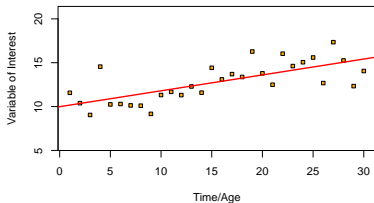
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Cross-sectional vs. longitudinal model

- Observations $(Y_{ij}, X_{ij1}, \dots, X_{ijp})^T$, for $i = 1, \dots, N \in \mathbb{N}$ and $n_i, p \in \mathbb{N}$
- Cross-sectional model ($n_i = 1$)

$$Y_{i1} = \beta_{CS} X_{i1} + \varepsilon_{i1} \quad (1)$$

- Longitudinal model ($n_i \in \mathbb{N}$)

$$Y_{ij} = \beta_{CS} X_{i1} + \beta_L (X_{ij} - X_{i1}) + \varepsilon_{ij} \quad (2)$$

→ for $j = 1$ the later model reduces to the former model, thus β_{CS} has the same interpretation in both models;

→ in addition, there is also β_L (a longitudinal dependence structure) parameter – its interpretation is quite straightforward when subtracting (1) from (2):

$$(Y_{ij} - Y_{i1}) = \beta_L (X_{ij} - X_{i1}) + (\varepsilon_{ij} - \varepsilon_{i1})$$

Cross-sectional vs. longitudinal interpretation

- ❑ **Cross-sectional interpretation of β_{CS}**

(averaging over subpopulations with the same values of X)

To estimate how individuals change over time with the cross-sectional data it needs to be assumed that the effects coincide $\Rightarrow \beta_{CS} = \beta_L$

- ❑ **Longitudinal interpretation of β_L**

(change within a specific subject per unit change of X within the subject)

No restriction in the form $\beta_{CS} = \beta_L$ is needed and longitudinal approaches are usually more powerful even in situations when $\beta_{CS} = \beta_L$

- ❑ **Population-specific interpretation vs. subject-specific interpretation**

(two different sources of variability that can be properly distinguished)

- ❑ **Associative vs. causal interpretation of the model**

(however, this is not the causal inference)

Borrowing power

- **Inference on β_{CS}** (marginal model)
 - *averaging individuals with one value of X and comparing with averaged individuals with another value of X – the estimated parameter $\hat{\beta}_{CS}$ stands for the expected/estimated change between subpopulations which corresponds to the unit change of X*

- **Inference on β_L** (hierarchical model)
 - *comparing a specific person's response at two distinct time points while X changes over time within the given subject – the estimated parameter $\hat{\beta}_L$ stands for the expected/estimated change (time development) within the subject which corresponds to the unit change of X over time (within the given subject)*

Borrowing power across subjects (sometimes possible, sometimes not)

Exploration of the longitudinal data

- ❑ The first step when analyzing (any) data is to perform a proper exploratory analysis...
- ❑ In case of longitudinal data structures, the exploratory becomes more complex...
 - ❑ exploratory of the mean structure
 - ❑ exploratory of the variance-covariance structure
 - ❑ exploratory of the between-subject dependence structure
 - ❑ exploratory of the subject-specific dependence structure

Question: What are common empirical/graphical tools to perform an exploratory analysis on longitudinal data (knowing or not knowing that the data are group-dependent?)