87) $X \sim N(\mu, \sigma^2)$ $\quad H_0: \mu = \mu_0$ $\qquad \tau = \mu, \ \psi = \sigma^2$ $\qquad$ Pr 63

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$$L(\mu, \sigma^2) = \frac{1}{(\sqrt{2\pi\sigma^2})^m} \exp\left\{ -\frac{1}{2\sigma^2} \sum (x_i - \mu)^2 \right\} \qquad \ell(\mu, \sigma^2) = c - \frac{m}{2} \log \sigma^2 - \frac{1}{2\sigma^2} \sum (x_i - \mu)^2$$

$$U(\mu, \sigma^2) = \Bigg( \underbrace{\frac{\sum (x_i - \mu)}{\sigma^2}}_{U_1}, -\frac{m}{2\sigma^2} + \frac{\sum (x_i - \mu)^2}{2\sigma^4} \Bigg) \qquad \hat{\mu} = \bar{X} \qquad \hat{\sigma}^2 = \frac{1}{m} \sum (x_i - \bar{x})^2$$

$$\hat{\theta} = (\hat{\mu}, \hat{\sigma}^2)$$

• pro $\tau = c_0$: $\quad \tilde{\sigma}^2 = \frac{1}{m} \sum (X_i - \mu_0)^2 \qquad \tilde{\theta} = (\mu_0, \tilde{\sigma}^2)$

$$-E \frac{\partial U}{\partial \theta^T}(\mu, \sigma^2) = \begin{pmatrix} +\frac{m}{\sigma^2} & 0 \\ 0 & \frac{m}{2\sigma^4} \end{pmatrix} = J_m(\theta) \qquad J(\theta) = \begin{pmatrix} \frac{1}{\sigma^2} & 0 \\ 0 & \frac{1}{2\sigma^4} \end{pmatrix}$$

$$J^{-1}(\theta) = \begin{pmatrix} \sigma^2 & 0 \\ 0 & 2\sigma^4 \end{pmatrix} \qquad J^{11}(\theta) = \sigma^2$$

• $LR := \cancel{2} \cdot \left[ \cancel{c} - \frac{m}{\cancel{2}} \log \hat{\sigma}^2 - \cancel{\frac{1}{2\hat{\sigma}^2} \sum (X_i - \bar{X})^2} - \cancel{c} + \frac{m}{\cancel{2}} \log \tilde{\sigma}^2 + \cancel{\frac{1}{2\tilde{\sigma}^2} \sum (X_i - \mu_0)^2} \right]$

$$= m \log \frac{\tilde{\sigma}^2}{\hat{\sigma}^2} = m \log \frac{\sum (X_i - \mu_0)^2}{\sum (X_i - \bar{X})^2}$$

• $W = m (\bar{X} - \mu_0)^2 (\hat{\sigma}^2)^{-1}$

• $R = \frac{1}{m} \left( \frac{\sum (X_i - \mu_0)}{\tilde{\sigma}^2} \right)^2 \tilde{\sigma}^2 = m \frac{(\bar{X} - \mu_0)^2}{\tilde{\sigma}^2}$ $\qquad$ Každý test zamítáme až $T_m > \chi^2_1(1-\alpha)$

88) $X \sim \log N(\mu, \sigma^2)$ $\quad H_0: \mu = \mu_0$ $\qquad \tau = \mu, \ \psi = \sigma^2$ $\qquad$ Pr 64

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$$\ell(\mu, \sigma^2) = c - \frac{m}{2} \log \sigma^2 - \frac{1}{2\sigma^2} \sum (\log x_i - \mu)^2$$

$$U = \left( \sum (\log x_i - \mu)/\sigma^2, \ -m/2\sigma^2 + \sum (\log x_i - \mu)^2/2\sigma^4 \right) \qquad \hat{\mu} = \frac{1}{m} \sum \log X_i \quad \hat{\sigma}^2 = \frac{1}{2} \sum (\log X_i - \hat{\mu})^2$$

$$\tilde{\sigma}^2 = \frac{1}{m} \sum (\log X_i - \mu_0)^2$$

$$J(\theta) = \begin{pmatrix} 1/\sigma^2 & 0 \\ 0 & 1/2\sigma^4 \end{pmatrix} \Rightarrow J^{11}(\theta) = \sigma^2$$

• $LR = \dots = m \log \left[ \frac{\sum (\log X_i - \mu_0)^2}{\sum (\log X_i - \hat{\mu})^2} \right]$

• $W = m (\hat{\mu} - \mu_0)^2 (\hat{\sigma}^2)^{-1}$

• $R = \frac{1}{m} \left[ \frac{\sum (\log X_i - \mu_0)}{\tilde{\sigma}^2} \right]^2 \cdot \tilde{\sigma}^2 = m \frac{(\hat{\mu} - \mu_0)^2}{\tilde{\sigma}^2}$ $\qquad$ zamítáme až $T_m > \chi^2_1(1-\alpha)$

89) $Y|N \sim Bi(\tilde{m}, p)$ pre $\tilde{m} = N$, $N \sim Po(\lambda)$ $\quad H_0: p = p_0 \quad \tau = p,\ \psi = \lambda$ $\quad$ Gn 69

96

$$L(p,\lambda) = \prod \binom{m_i}{y_i} p^{y_i}(1-p)^{m_i-y_i} \frac{\lambda^{m_i} e^{-\lambda}}{m_i!} = c \cdot p^{\sum y_i} (1-p)^{\sum m_i - \sum y_i} \lambda^{\sum m_i} e^{-m\lambda}$$

$$\ell(p,\lambda) = c + \sum y_i \log p + \left(\sum m_i - \sum y_i\right) \log(1-p) + \sum m_i \log \lambda - m\lambda$$

$$U(p,\lambda) = \left( \frac{\sum y_i}{p} - \frac{\sum m_i - \sum y_i}{1-p},\ \frac{\sum m_i}{\lambda} - m \right) \qquad \hat{\lambda} = \frac{\sum m_i}{m} \qquad \hat{p} = \frac{\sum y_i}{\sum m_i}$$

$\hat{\lambda}$ nezávisí na $p \Rightarrow \tilde{\lambda} = \hat{\lambda}$

$$J(p,\lambda) = \begin{pmatrix} \frac{\lambda}{p} + \frac{\lambda}{1-p} & 0 \\ 0 & \frac{1}{\lambda} \end{pmatrix} \qquad J^{11}(\theta) = \frac{1/1}{\frac{(1-p)\lambda + p\lambda}{p(1-p)}} = \frac{p(1-p)}{\lambda}$$

• $$LR = 2\Big[ \cancel{c} + \sum Y_i \log \hat{p} + \sum (N_i - Y_i) \log(1-\hat{p}) + \cancel{\sum N_i \log \hat{\lambda}} - \cancel{m\hat{\lambda}} - \cancel{c} - \sum Y_i \log p_0 - \sum (N_i - Y_i) \log(1-p_0) - \cancel{\sum N_i \log \hat{\lambda}} + \cancel{m\hat{\lambda}} \Big]$$
$$= 2\Big[ \sum Y_i \log \hat{p}/p_0 + \sum (N_i - Y_i) \log[(1-\hat{p})/(1-p_0)] \Big]$$

• $$W = m(\hat{p} - p_0)^2 \left[ \frac{\hat{p}(1-\hat{p})}{\hat{\lambda}} \right]^{-1} = m \frac{(\hat{p}-p_0)^2}{\hat{p}(1-\hat{p})} \cdot \hat{\lambda}$$

• $$R = \frac{1}{m} \left( \frac{\sum Y_i}{p_0} - \frac{\sum (N_i - Y_i)}{1-p_0} \right)^2 \frac{p_0(1-p_0)}{\hat{\lambda}} = \frac{1}{m} \left( \frac{\sum Y_i - \cancel{p_0 \sum Y_i} - p_0 \sum N_i + \cancel{p_0 \sum Y_i}}{p_0(1-p_0)} \right)^2 \frac{p_0(1-p_0)}{\hat{\lambda}}$$
$$= \frac{1}{m} \left( \sum N_i \left( \frac{\sum Y_i}{\sum N_i} - p_0 \right) \right)^2 \frac{1}{\hat{\lambda} p_0 (1-p_0)}$$

zamietam ak $T_n > \chi^2_1(1-\alpha)$

97

90) $X \sim M(1, p_1, p_2, p_3, p_4)$ $\quad$ Pr 67+1

→ • štandardná parametrizácia $(p_1, p_2, p_3, p_4)^T = p$ $\qquad y_j = \sum_{i=1}^{m} x_{ij}$ (počet $j$ v súbore)

1) $L(p) = \prod p_j^{y_j}$ $\qquad \ell(p) = \sum y_j \log p_j$ $\qquad p_1 = \tau \quad (p_2, p_3, p_4) = \psi \quad H_0: p_1 = 1/4$

$\hat{p}_j = y_j/m$ $\quad$ pre $\tilde{p}$ maximalizujú $\sum y_j \log p_j$ na podm. $p_1 = 1/4$, $\sum_{j=2}^{4} p_j = \frac{3}{4}$

tj $\quad f(p_2, p_3, p_4, \lambda) = y_1 \log 1/4 + y_2 \log p_2 + y_3 \log p_3 + y_4 \log p_4 + \lambda\left(\frac{3}{4} - \sum p_j\right)$

$$\partial/\partial p_j\, f = y_j/p_j - \lambda = 0 \qquad \partial/\partial\lambda\, f = 3/4 - \sum p_j = 0$$
$$\Rightarrow p_j = y_j/\lambda \quad \rightarrow \quad 3/4 - 1/\lambda \sum y_j = 0 \Rightarrow \lambda = 4/3 \sum y_j$$
$$\Rightarrow \tilde{p}_j = \frac{y_j}{\sum_{j=2}^{4} y_j} \cdot \frac{3}{4} \qquad j = 2,3,4$$

• $$LR = 2\left( \sum_{j=1}^{4} Y_j \log(Y_j/m) - Y_1 \log 1/4 - \sum_{j=2}^{4} Y_j \log\left( \frac{Y_j \cdot \frac{3}{4}}{\sum_{i=2}^{4} Y_i} \right) \right)$$

• Fisher. inf. nemôžeme počítať lebo nemáme ako zahrnúť do úvahy podm. $\sum_{j=1}^{4} p_j = 1$

→ • parametrizácia $(p_1, p_2, p_3, 1-p_1-p_2-p_3)$ $\qquad p = (p_1, p_2, p_3)^T$

$$L(p) = \prod_{j=1}^{3} p_j^{y_j} \cdot (1-p_1-p_2-p_3)^{m-y_1-y_2-y_3}$$ Mathematica $\qquad \hat{p}_j = y_j/m$

$$1 - \sum_{j=1}^{3} \hat{p}_j = y_4/m$$

$$\tilde{p} = \left( 1/4,\ 3/4 \frac{y_2}{m-y_1},\ 3/4 \frac{y_3}{m-y_1} \right)^T \overset{\text{zmena}}{\Longrightarrow} 1 - \sum_{j=1}^{3} \tilde{p}_j = \frac{3/4\, Y_4}{\sum_{i=2}^{4} Y_i}$$

• $$LR = 2\left( \sum_{j=1}^{3} Y_j \log(Y_j/m) + Y_4 \log(Y_4/m) - Y_1 \log 1/4 - \sum_{j=2}^{3} Y_j \log\left( \frac{Y_j\, 3/4}{m - Y_1} \right) - Y_4 \log\left( \frac{3/4\, Y_4}{m - Y_1} \right) \right)$$

91) z Př 90 a R skriptu dostáváme $\hat{p} \doteq (0{,}240;\ 0{,}258;\ 0{,}267;\ 0{,}235)$

(98) a) $LR = 60{,}08$ $\quad H_0: p_1 = 1/4$ $\quad \chi^2_1(1-\alpha) = 3{,}84$ $\quad$ p-val $= 9 \cdot 10^{-15}$ $\Rightarrow$ zamítáme

b) $H: p_1 = p_2$ $\quad LR = 76{,}41$ $\quad$ p-val $\doteq 2 \cdot 10^{-18}$ $\Rightarrow$ zamítáme

c) $H: p_3 = 1{,}1\, p_1$ $\quad LR = 1{,}39$ $\quad$ p-val $= 0{,}239$ $\Rightarrow$ nezamítáme

92) $(X_i, Y_i)^T \sim$ náh. výběr kde $Y_i \mid X_i \sim N(\beta_0 + \beta_1 x_i, \sigma^2)$ $\quad X_i$ nezáv. na $\beta, \sigma^2$

(99) pro $\sigma = 1$ jsme řešili v Př 47 (Př 22)

$H_0: \beta_1 = 0 \quad H_1: \beta_1 \neq 0$

$$L(\beta, \sigma^2) = c \cdot \prod \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{1}{2\sigma^2}(y_i - \beta_0 - \beta_1 x_i)^2\right\} = c(\sigma^2)^{-n/2} \exp\left\{-\frac{1}{2\sigma^2}\sum (y_i - \beta_0 - \beta_1 x_i)^2\right\}$$

$$\ell(\beta, \sigma^2) = c - \frac{n}{2}\log \sigma^2 - \frac{1}{2\sigma^2}\sum (y_i - \beta_0 - \beta_1 x_i)^2$$

$$\frac{\partial \ell}{\partial \sigma^2} = -\frac{n}{2\sigma^2} + \frac{1}{2\sigma^4}\sum (y_i - \beta_0 - \beta_1 x_i)^2 \overset{!}{=} 0 \Rightarrow \hat{\sigma}^2 = \frac{1}{n}\sum (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i)^2$$

$$\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X} \qquad \hat{\beta}_1 = \frac{\sum (Y_i - \bar{Y})^2 (x_i - \bar{X})^2}{\sum (x_i - \bar{x})^2}$$

pro $\hat{\beta}_0$ a $\hat{\beta}_1$ v Př 47 (Př 22) (Dupač, Hušková) Věta 7.1

na $H_0$: $\ell(\beta_0, \sigma^2) = c - \frac{n}{2}\log \sigma^2 - \frac{1}{2\sigma^2}\sum (y_i - \beta_0)^2$

$\Rightarrow$ standardní normální model $\Rightarrow$ $\tilde{\beta}_0 = \bar{Y}$ $\quad \tilde{\sigma}^2 = \frac{1}{n}\sum (Y_i - \tilde{\beta}_0)^2$ $\quad \tilde{\beta}_1 = 0$

$$\bullet\ LR = 2\left[\cancel{c} - \frac{n}{2}\log \hat{\sigma}^2 - \underbrace{\frac{1}{2\hat{\sigma}^2}\sum (Y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i)^2}_{n/2} - \cancel{c} + \frac{n}{2}\log \tilde{\sigma}^2 + \underbrace{\frac{1}{2\tilde{\sigma}^2}\sum (Y_i - \tilde{\beta}_0)^2}_{n/2}\right]$$

$$= \frac{2n}{2}\left[\log\left[\frac{\tilde{\sigma}^2}{\hat{\sigma}^2}\right]\right]$$

$$= n \log \frac{\sum (Y_i - \tilde{\beta}_0)^2}{\sum (Y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i)^2}$$

zamítáme když $LR > \chi^2_1(1-\alpha)$

93) náhodný výběr $(X_i, Y_i)$ jako v Př 41 kde

(100) $P(Y = 1 \mid X = x) = \dfrac{e^{\alpha + \beta x}}{1 + e^{\alpha + \beta x}}$ $\qquad P(Y = 0 \mid X = x) = 1 - P(Y = 1 \mid X = x)$

$$\ell(\alpha, \beta) = \sum y_i(\alpha + \beta x_i) - \sum \log(1 + e^{\alpha + \beta x_i})$$

$\hat{\alpha}, \hat{\beta}$ numericky v R skriptu

$$U(\alpha, \beta) = \begin{pmatrix} \sum y_i - \sum e^{\alpha + \beta x_i}/(1 + e^{\alpha + \beta x_i}) \\ \sum x_i y_i - \sum x_i e^{\alpha + \beta x_i}/(1 + e^{\alpha + \beta x_i}) \end{pmatrix} \} U_2$$

$\bullet$ na $H_0: \beta = 0$ $\qquad \ell(\alpha) = \sum y_i \alpha - \sum \log(1 + e^{\alpha})$

$$U(\alpha) = \sum y_i - \sum \frac{e^{\alpha}}{1 + e^{\alpha}} \overset{!}{=} 0 \Rightarrow \bar{Y} = e^{\alpha}/(1 + e^{\alpha}) \qquad \tilde{\alpha} = \log \frac{\bar{Y}}{1 - \bar{Y}}, \quad \tilde{\beta} = 0$$

$$LR = 2\left[\sum Y_i(\hat{\alpha} + \hat{\beta} x_i) - \sum \log(1 + e^{\hat{\alpha} + \hat{\beta} x_i}) - \sum Y_i \tilde{\alpha} + \sum \log(1 + e^{\tilde{\alpha}})\right]$$

Informační matice: z Př 41 máme pro $X = \begin{pmatrix} 1 & x_1 \\ \vdots & \vdots \\ 1 & x_n \end{pmatrix}$ a $W = \operatorname{diag}\Bigg(\underbrace{\frac{e^{\alpha + \beta x_i}}{(1 + e^{\alpha + \beta x_i})^2}}_{w_i}\Bigg)$

že $J_n(\alpha, \beta) = X'WX = \begin{pmatrix} \sum w_i & \sum w_i x_i \\ \sum w_i x_i & \sum w_i x_i^2 \end{pmatrix}$

$\tau = \beta$, $H_0: \beta = 0$ – pro $J^{11}$ prvek $(2,2)$ matice $[J_n(\alpha, \beta)/n]^{-1}$ $\qquad$ Stejné porovn. $\sim \chi^2_1(1-\alpha)$

$$R = \frac{1}{n}\left[U_2(\tilde{\alpha}, 0)\right]^2 J^{11}(\tilde{\alpha}, 0) \qquad W = n\left((\hat{\alpha}, \hat{\beta}) - (\hat{\alpha}, 0)\right)^2 / J^{11}(\hat{\alpha}, \hat{\beta})$$

90) pokrač. $\Rightarrow$ LR testy sú v oboch parametrizáciách rovnaké.

(94) Fisherova informácia Mathematica

$$J(\mathbb{p}) = \begin{pmatrix} 1/p_1 + 1/(1-p_1-p_2-p_3) & 1/(1-p_1-p_2-p_3) & 1/(1-p_1-p_2-p_3) \\ \cdot & 1/p_2 + 1/(1-p_1-p_2-p_3) & \cdot \\ \cdot & \cdot & 1/p_3 + 1/(1-p_1-p_2-p_3) \end{pmatrix}$$

$$J^{-1}(\mathbb{p}) = \begin{pmatrix} p_1(1-p_1) & -p_1p_2 & -p_1p_3 \\ -p_1p_2 & p_2(1-p_2) & -p_2p_3 \\ -p_1p_3 & -p_2p_3 & p_3(1-p_3) \end{pmatrix}$$

$$W = m\left(Y_1/m - p_0\right)^2\left[\hat{p}_1(1-\hat{p}_1)\right]^{-1} = \left[\frac{\sqrt{m}(\hat{p}_1 - p_0)}{\sqrt{\hat{p}_1(1-\hat{p}_1)}}\right]^2$$

b) v štandardnej parametrizácii $H_0: p_1 = p_2$ $\quad \ell(\mathbb{p}) = \sum y_j \log p_j$

na $H_0$: $\ell(p_2, p_3, p_4) = (y_1+y_2)\log p_2 + y_3 \log p_3 + y_4 \log p_4$ $\Rightarrow$ ako v a) na podm. $2p_2 + p_3 + p_4 = 1$

$$\tilde{\mathbb{p}} = \left(\frac{Y_1+Y_2}{2m}, \frac{Y_1+Y_2}{2m}, \frac{Y_3}{m}, \frac{Y_4}{m}\right)$$

$$\partial f/\partial p_j = y_j/p_j - \lambda = 0, \quad j>2$$
$$= (y_1+y_2)/p_j - 2\lambda = 0 \quad j=2 \qquad \Rightarrow \quad p_j = \begin{cases} (y_1+y_2)/2\lambda & j=2 \\ y_j/\lambda & j>2 \end{cases}$$

$$\partial f/\partial \lambda \Rightarrow (y_1+y_2)/2\lambda \cdot 2 + y_3/\lambda + y_4/\lambda = 1 \Rightarrow \lambda = \sum_{i=1}^{4} y_i$$

$$\tilde{p}_2 = (y_1+y_2)/2\cdot\sum y_i = \frac{y_1+y_2}{2m} \qquad \tilde{p}_3 = y_3/m \qquad \tilde{p}_4 = y_4/m$$

- $$LR = 2\left[\sum_{j=1}^{4} Y_j \log Y_j/m - Y_1\log\left(\frac{Y_1+Y_2}{2m}\right) - Y_2\log\left(\frac{Y_1+Y_2}{2m}\right) - Y_3\log\frac{Y_3}{m} - Y_4\log\frac{Y_4}{m}\right]$$
$$= 2\left[Y_1\log\frac{Y_1}{m} + Y_2\log\frac{Y_2}{m} - (Y_1+Y_2)\log\frac{Y_1+Y_2}{2m}\right]$$

c) ako v b) $\quad \ell(p_1, p_2, p_4) = y_1\log p_1 + y_2 \log p_2 + y_3 \log 1{,}1p_1 + y_4 \log p_4 \quad \left(+\lambda(2{,}1p_1 + p_2 + p_4 - 1)\right)$

$$\partial f/\partial p_j = \begin{cases} y_j/p_j - \lambda, & j = 2,4 \\ (y_1+y_3)/p_1 - 2{,}1\cdot\lambda & j=1 \end{cases} \qquad p_j = \begin{cases} y_j/\lambda & j=2,4 \\ \dfrac{(y_1+y_3)}{2{,}1\lambda} & j=1 \end{cases}$$

$$\partial f/\partial\lambda \Rightarrow \lambda = \sum_{i=1}^{4} y_i = m \qquad (\tilde{p}_1, \tilde{p}_2, \tilde{p}_4) = \left(\frac{Y_1+Y_3}{2{,}1\cdot m}, \frac{Y_2}{m}, \frac{Y_4}{m}\right)$$

$$\tilde{p}_3 = 1{,}1\cdot\tilde{p}_1 = \frac{1{,}1}{2{,}1}\cdot\frac{Y_1+Y_3}{m}$$

- $$LR = 2\left[\sum_{i=1}^{4} Y_j\log\frac{Y_j}{m} - Y_1\cdot\log\frac{Y_1+Y_3}{2{,}1m} - Y_2\log\frac{Y_2}{m} - Y_3\log\left[\frac{1{,}1}{2{,}1}\left(\frac{Y_1+Y_3}{m}\right)\right] - Y_4\log\frac{Y_4}{m}\right]$$

zamietam ak $LR > \chi^2_1(1-\alpha)$

93) cont. b) výsledky z R skriptu

(100)

$LR = 1{,}138$
$R = 1{,}078$ porovnávam s $\chi^2_1(0{,}95) = 3{,}84$ $\Rightarrow$ p-value $0{,}286$, $0{,}299$, $0{,}323$
$W = 0{,}979$

nezamietam $H_0: \beta = 0$ voči $H_1: \beta \neq 0$.
On interval spoľahlivosti vyjdem z $W = \sqrt{m}\,\hat\beta / \sqrt{J^{11}(\hat\alpha,\hat\beta)} \overset{as}{\sim} N(0,1)$

$$\left[\hat\beta \mp u_{1-\alpha/2} \sqrt{J^{11}(\hat\alpha,\hat\beta)}/\sqrt{m}\right] \approx [-0{,}055;\ 0{,}168]$$

(101)

94) podobne ako v Pr 78 (87) $X \sim R(0,1)$ $Y|X \sim Exp(\lambda(\alpha,\beta,x))$ pre $\lambda(\alpha,\beta,x) = e^{\alpha+\beta x}$

a) $H_0: \beta = 0$ $\quad L(\alpha,\beta) = \prod e^{\alpha+\beta x_i} \exp\{-e^{\alpha+\beta x_i} \cdot y_i\} = e^{\alpha m + \beta \sum x_i} \exp\{-\sum y_i e^{\alpha+\beta x_i}\}$

$$\ell(\alpha,\beta) = \alpha m + \beta \sum x_i - \sum y_i e^{\alpha+\beta x_i}$$

$$U = \left(m - \sum y_i e^{\alpha+\beta x_i},\ \sum x_i - \sum x_i y_i e^{\alpha+\beta x_i}\right)$$ $\hat\alpha, \hat\beta$ iba numericky

$$\frac{\partial U}{\partial \theta'} = \begin{pmatrix} -\sum y_i e^{\alpha+\beta x_i} & -\sum y_i x_i e^{\alpha+\beta x_i} \\ -\sum y_i x_i e^{\alpha+\beta x_i} & -\sum y_i x_i^2 e^{\alpha+\beta x_i} \end{pmatrix} \qquad J(\alpha,\beta) = \begin{pmatrix} 1 & 1/2 \\ 1/2 & 1/3 \end{pmatrix}$$

$$E[X^k Y e^{\alpha+\beta X}] = E[E(X^k Y e^{\alpha+\beta X} | X)] = E\, X^k \cancel{e^{\alpha+\beta X}} \cdot \frac{1}{\cancel{e^{\alpha+\beta X}}} = EX^k = \begin{cases} 1 & k=0 \\ 1/2 & k=1 \\ 1/3 & k=2 \end{cases}$$

$$\tau = \beta \qquad J^{-1} = 12 \cdot \begin{pmatrix} 1/3 & -1/2 \\ -1/2 & 1 \end{pmatrix} = \begin{pmatrix} 4 & -6 \\ -6 & \boxed{12} \end{pmatrix} \qquad J^{11} = 12$$

- na $H_0: \beta = 0$ $\quad \ell(\alpha) = \alpha m - \sum y_i e^{\alpha}$
  $U(\alpha) = m - \sum y_i e^{\alpha} \Rightarrow \tilde\alpha = \log \frac{m}{\sum Y_i} = -\log \bar{Y}$
- $LR = 2\left[\hat\alpha m + \hat\beta \sum X_i - \sum Y_i e^{\hat\alpha + \hat\beta X_i} - \tilde\alpha m + \sum Y_i e^{\tilde\alpha}\right]$
- $R = \frac{1}{m}\left(\sum X_i - \sum X_i Y_i e^{\tilde\alpha}\right)^2 \cdot 12$ $\quad$ porovnávam s $\chi^2_1(1-\alpha)$
- $W = m(\hat\beta - \beta_0)^2/12 = m\hat\beta^2/12$

b) pre obecné $X$ musím iba $J(\alpha,\beta) = \begin{pmatrix} EX^0 & EX^1 \\ EX^1 & EX^2 \end{pmatrix}$ inak je všetko rovnaké
ale odhadnem $J$ pozorovanou inf. maticou

(102)

95) pokračovanie multinomického rozdelenia: $p = (p_{11}, p_{12}, p_{21}, p_{22})^T$ ako v Pr 90

$H_0: p_{12} = p_{21}$, Pr 90 b) $p_{21}$ a $p_{12}$ ako $p_1$ a $p_2$ tam
$H_1: p_{12} \neq p_{21}$

$$\hat{p} = \left(\frac{Y_{11}}{m}, \frac{Y_{12}}{m}, \frac{Y_{21}}{m}, \frac{Y_{22}}{m}\right)^T \qquad \tilde{p} = \left(\frac{Y_{11}}{m}, \frac{Y_{12}+Y_{21}}{2m}, \frac{Y_{12}+Y_{21}}{2m}, \frac{Y_{22}}{m}\right)^T$$

$$LR = 2\left[Y_{12} \log \frac{Y_{12}}{m} + Y_{21} \log \frac{Y_{21}}{m} - (Y_{12}+Y_{21}) \log \frac{Y_{12}+Y_{21}}{2m}\right]$$ zamietam $> \chi^2_1(1-\alpha)$

(103)

96) zložitejšia situácia ako v Pr 60 a Pr 79 $(X_i', Y_i)'$ nez. výber

$Y | X = x \sim Po(e^{\alpha+\beta' x})$, $X$ nezávisí na $\alpha, \beta$ $\qquad H_0: \beta = 0_q \quad H_1: \beta \neq 0_q$

$$L(\alpha,\beta) = \prod \lambda(x_i)^{y_i} \cdot \frac{e^{-\sum \lambda(x_i)}}{\prod y_i!} \qquad \ell(\alpha,\beta) = \sum y_i \log \lambda(x_i) - \sum \lambda(x_i) + c$$

$$\lambda(x) = e^{\alpha+\beta' x} \qquad \frac{\partial}{\partial\theta} \lambda(x) = (\lambda(x),\ x\lambda(x))$$

$$U(\alpha,\beta) = \sum \frac{y_i}{\lambda(x_i)} \begin{pmatrix}\lambda(x_i)\\ x_i\lambda(x_i)\end{pmatrix} - \sum \begin{pmatrix}\lambda(x_i)\\ x_i\lambda(x_i)\end{pmatrix} = \sum \begin{pmatrix} y_i \\ y_i x_i\end{pmatrix} - \sum \begin{pmatrix}\lambda(x_i)\\ x_i\lambda(x_i)\end{pmatrix}$$

$\hat\alpha, \hat\beta$ numericky

$$\frac{\partial}{\partial\theta'} U = \begin{pmatrix} -\sum\lambda(x_i) & -\sum x_i'\lambda(x_i) \\ -\sum x_i\lambda(x_i) & -\sum x_i x_i'\lambda(x_i)\end{pmatrix} \qquad J(\alpha,\beta) = \begin{pmatrix} E\lambda(X) & EX'\lambda(X)\\ EX\lambda(X) & EXX'\lambda(X)\end{pmatrix}$$

$J^{11}(\alpha,\beta)$ = podmatica $(2..(q+1), 2..(q+1))$ matice $J(\alpha,\beta)$.

za $H_0: \beta = 0$ $\quad \ell(\alpha) = \sum \alpha y_i - n e^{\alpha} \quad U(\alpha) = \sum y_i - n e^{\alpha} \Rightarrow \tilde\alpha = \log \bar Y$, $\tilde\beta = 0$

- $LR = 2\left[\sum Y_i \log e^{\hat\alpha+\hat\beta' x_i} - \sum e^{\hat\alpha+\hat\beta' x_i} - \sum \log\bar Y \cdot Y_i + n\bar Y\right]$
- $R = \frac{1}{n}\left(\sum(Y_i X_i - X_i e^{\tilde\alpha})\right)' \cdot J^{11}(\tilde\alpha,0)\left(\sum(Y_i X_i - X_i e^{\tilde\alpha})\right)$
- $W = n(\hat\beta - 0)' J^{11}(\hat\alpha,\hat\beta)^{-1}(\hat\beta - 0)$ $\qquad$ porovnám s $\chi^2_q(1-\alpha)$

b) výsledky z R skriptu

| | | p-value |
|---|---|---|
| $LR = 11{,}21$ | | $0{,}003$ |
| $R = 10{,}94$ | porovnávam s $\chi^2_2(1-\alpha) = 5{,}99$ | $0{,}004$ |
| $W = 8{,}91$ | | $0{,}011$ |

zamietam $H_0: \beta = (0,0)$

interval spoľahlivosti pre $\beta_1$: cez $W$ a jej 1. marginálne rozdelenie $\approx [-0{,}75 ; 1{,}44]$.