97) $X_1 \dots X_n \sim Exp(\lambda)$ $\quad P(X_{(1)} \le t) = 1-(1-P(X_1 \le t)^{n})^{n} = 1-(1-(1-e^{-n\lambda x}))^{n} = 1 - e^{-n\lambda x} \sim Exp(n\lambda)$

$L(\lambda) = n\lambda e^{-n\lambda x}$ $\quad \ell(\lambda) = c + \lg n - n\lambda x + \lg \lambda$ $\quad U(\lambda) = \frac{1}{\lambda} - nx \overset{!}{=} 0$

$\frac{1}{\lambda} = nx \Rightarrow \hat{\lambda} = \frac{1}{n X_{(1)}}$ $\quad P(n X_{(1)} \le t) = 1 - e^{-n\lambda t/n} = 1 - e^{-\lambda t}$

$\Rightarrow n X_{(1)} \sim Exp(\lambda)$ a $\hat{\lambda} \sim [Exp(\lambda)]^{-1}$ a nie je konzistentný odhad.

98) a) $(X_i, Y_i) \sim R$ (kruh s polomerom θ) $\quad f(x,y) = \frac{1}{\pi\theta^2} I[x^2+y^2 \le \theta^2]$

$L(\theta) = \prod \frac{1}{\pi\theta^2} I[x_i^2+y_i^2 \le \theta^2] = \left(\frac{1}{\pi\theta^2}\right)^n I[x_i^2+y_i^2 \le \theta^2 \ \forall i]$

maximalizujem pre čo najmenšie θ za podm. $x_i^2+y_i^2 \le \theta^2 \Rightarrow \hat{\theta} = \max_i \sqrt{X_i^2+Y_i^2}$

b) $(X_i, Y_i) \sim R([-\theta,\theta]^2)$ $\quad f(x,y) = \frac{1}{4\theta^2} I[x_i \in [-\theta,\theta], y_i \in [-\theta,\theta]]$

$\Rightarrow \hat{\theta} = \max_i \{|X_i| \vee |Y_i|\}$

99) n = pre rodiny s m deťmi $\quad X_1 \dots X_n \sim Bi(m,p)$

a) $L(p) = \prod_{i=1}^{n} \binom{m}{x_i} p^{x_i} (1-p)^{m-x_i} = \prod \binom{m}{x_i} \cdot p^{\sum x_i} (1-p)^{nm - \sum x_i}$

$\ell(p) = \sum \lg \binom{m}{x_i} + \sum x_i \lg p + (nm - \sum x_i) \lg (1-p)$

$U(p) = \sum x_i / p + - \frac{nm - \sum x_i}{1-p} = 0$

$\sum x_i - p\sum x_i - nmp + p\sum x_i = 0$ $\quad$ Rskript

$\hat{p} = \frac{1}{nm} \sum x_i \Rightarrow \hat{p} = 0{,}514$

b) rodiny s 2 alebo 6 deťmi – rozn. $Bi(2,p)$ a $Bi(6,p)$ s neurn. rozn.

$\binom{x_i}{c_i}$ $\quad x_i$ – počet chlapcov z m detí $\quad$ náhodný jav $\quad n_1 = \sum c_i$ (počet 6-čl. rodín)

$c_i = I[m=6]$ inak $m=2$

$L(p) = \prod_{i=1}^{n} \left[\binom{2}{x_i} p^{x_i} (1-p)^{2-x_i}\right]^{1-c_i} \left[\binom{6}{x_i} p^{x_i} (1-p)^{6-x_i}\right]^{c_i} = k \cdot p^{\sum x_i c_i + \sum x_i (1-c_i)} \cdot (1-p)^{\sum (2-x_i)(1-c_i) + \sum (6-x_i) c_i}$

$\ell(p) = k + \sum x_i \lg p + (\sum (2-x_i) + \sum (6-x_i)) (1-p)$

(2-čl. rod. … 6-čl.)

$U(p) = \frac{\sum x_i}{p} + - \frac{\sum (2-x_i) + \sum (6-x_i)}{1-p} \overset{!}{=} 0$ $\quad$ (čitateľ $= a$)

$\sum x_i - \sum x_i p - ap = 0$

$p(n) = \sum x_i$

$\hat{p} = \frac{\sum x_i}{n} \Rightarrow \hat{p} = 0{,}514$

a = celkový počet detí – celkový počet chlapcov $= $ celkový počet dievčat $= n - \sum x_i$ (n … celk. poč. detí)

$\hat{p}$ = celk. poč. chlapcov / celk. poč. detí