Regularity theory for elliptic and parabolic systems and problems in continuum mechanics

Workshop, 30th April – 3rd May 2014, Telč

Schedule

Thursday, May 1

Time	Speaker	Title
8:00		Breakfast
8:50		Opening
9:00	Emilio Acerbi	An isoperimetric problem with applications to diblock copolymer melts
10:00	Maria Specovius-Neugebauer	Some regularity results for plasticity problems
10:30		Coffee & Refreshment
11:00	Nicola Fusco	Motion of elastic thin films by anisoptropic surface diffusion with curvature regularization
12:00	Mikhail Surnachev	On density of smooth functions in weighted Sobolev-Orlicz spaces with variable exponent
12:30		Lunch
15:30		Coffee & Refreshment
16:00	Lars Diening	Lipschitz truncation
17:10	Jakub Tichý	Higher integrability of generalized Stokes system under perfect slip boundary conditions
17:30		Break
17:45	Alexander Ukhlov	Conformal composition operators and Brennan's conjecture
18:15	Lenka Slavíková	Compactness of higher-order Sobolev embeddings
19:00		Dinner

Friday, May 2

Time	Speaker	Title
8:00		Breakfast
9:00	Stanislav Hencl	Diffeomorphic Approximation of $W^{1,1}$ Planar Sobolev Homeomorphisms
10:00	Šárka Nečasová	On the existence of weak solution to the coupled fluid-structure interaction problem for non-Newtonian shear-dependent fluid
10:30		Coffee & Refreshment
11:00	Giuseppe Mingione	Update on nonlinear potential theory
12:00	Sebastian Schwarzacher	BMO estimates for degenerate parabolic systems
12:30		Lunch
15:15		Coffee & Refreshment
15:45	Tuomo Kuusi	A quantitative modulus of continuity for the two-phase Stefan problem
16:45	Francesca Crispo	High regularity results of solutions to modified p-Navier-Stokes equations
17:15		Break
17:30	Katarzyna Ewa Mazowiecka	Conditional regularity for p-parabolic systems with critical right hand side
18:00	Jan Burczak	Regularity of evolutionary symmetric p-Laplacian
18:30	Paweł Subko	Remark on transport equation with $b \in BV$ and $\mathrm{div}_x b \in BMO$
19:15		Dinner

Saturday, May 3

Time	Speaker	Title
8:00		Breakfast
9:00	Andrea Cianchi	Global gradient estimates in elliptic problems under minimal data and domain regularity
10:00	Martin Kalousek	Homogenization of a non-Newtonian flow through a porous medium
10:30		Coffee & Refreshment
11:00	Mark Steinhauer	On Hölder continuity of solution to elliptic systems & variational integrals
12:00	Werner Varnhorn	On local strong solutions of the non homogeneous Navier-Stokes equations
12:30		Closing
12:45		Lunch
13:45		Departure

Emilio Acerbi, An isoperimetric problem with applications to diblock copolymer melts. Diplock copolymers are new engineerable materials with interesting mechanical properties given by their intrinsic periodic micro/macrostructure. Among the energies proposed to model them, a widely used one is the Ohta-Kawasaki functional, which contains a nonlocal term related to medium-range interactions. Our approach shifts the problem to a quantitative isoperimetric inequality, from which we deduce information on the strict minimality and stability of certain critical configurations.

Andrea Cianchi, Global gradient estimates in elliptic problems under minimal data and domain regularity. This talk is devoted to some results, in collaboration with V.Maz'ya, on global integrability properties of the gradient of solutions to boundary value problems for nonlinear elliptic equations (or systems, in some cases) in divergence form. Minimal assumptions on the regularity of the ground domain and of the prescribed data for a certain gradient bound are pursued. A distinctive feature of our approach is in the derivation of estimates which are flexible enough to be applied in the proof of gradient bounds for a wide choice of norms. Most of the relevant estimates are formulated in terms of point-wise inequalities for the distribution function of the length of the gradient, or, equivalently, for its decreasing rearrangement. With this tool at disposal, global bounds for any rearrangement invariant norm of the gradient of solutions to either Dirichlet or Neumann boundary value problems are simply reduced to one-dimensional inequalities for Hardy type operators.

Lars Diening, Lipschitz truncation. The approximation of Sobolev functions by Lipschitz functions can easily be done by convolution. However, this will change the function on the whole domain. The Lipschitz truncation technique allows to construct an approximation that differs from the original only on a small set. The idea goes back to Acerbi and Fusco in 1988, who both also participate at this conference. We explain the Lipschitz truncation technique in detail and apply it to stationary and unsteady problems.

Nicola Fusco, Motion of elastic thin films by anisoptropic surface diffusion with curvature regularization. We discuss time existence for a surface diffusion evolution equation with curvature regularization in the context of epitaxially strained films. This is achieved by implementing a minimizing movement scheme, which is hinged on the H^{-1}-gradient flow structure underpinning the evolution law. Long-time behavior and Lyapunov stability in the case of initial data close to a flat configuration are also addressed.

Stanislav Hencl, Diffeomorphic Approximation of $W^{1,1}$ Planar Sobolev Homeomorphisms. Well-known Evans-Ball open problem ask the following: Suppose that f is a homeomorphism from a subset of $\mathbb{R}^n$ $\mathbb{R}^n$ in the Sobolev space $W^{1,p}$. Can we then find a sequence of piecewise affine or smooth homeomorphisms f_k such that f_k converge to f in the Sobolev norm? The motivation for this problem comes from regularity problem in Nonlinear Elasticity and also from numerical approximations. We show that given a $W^{1,1}$ Sobolev homeomorphism in the plane we can indeed find a sequence of smooth approximations. This is a joint result with Aldo Pratelli.

Tuomo Kuusi, A quantitative modulus of continuity for the two-phase Stefan problem. We will derive the quantitative modulus of continuity

$$\omega(r) = \left[p + \ln\left(\frac{r_0}{r}\right)\right]^{-\alpha}, \qquad \alpha \equiv \alpha(n,p) := \frac{p}{n+p}, \qquad p < n,$$

for solutions of the p-degenerate two-phase Stefan problem. Even in the classical case $p = 2$, this represents a twofold improvement with respect to the early 1980's state-of-the-art results by Caffarelli-Evans and DiBenedetto, in the sense that we discard one logarithm iteration and obtain an explicit value for the exponent $\alpha(n,p)$. This is a joint work with P. Baroni and J.-M. Urbano.

Giuseppe Mingione, Update on nonlinear potential theory.

Mark Steinhauer, On Hölder continuity of solution to elliptic systems & variational integrals. We consider minimizers to a variational integral

$$J(u) := \int_\Omega F(u, \nabla u) - f \cdot u dx$$

or weak solution to the corresponding Euler-Lagrange equations in $W_0^{1,p}(\Omega;\mathbb{R}^N)$ setting with F having p-growth and coercivity with respect to the second variable. It is well known that generally such solutions for such nonlinear systems of PDE's or minimizers to $J(u)$ may not be smooth and even more they can be discontinuous or unbounded. Up to our best knowledge, the only known result when at least the Hölder continuity of a solution/minimizer can be shown is the so-called Uhlenbeck case $F(\nabla u) \sim |\nabla u|^p$ for large $|\nabla u|$ and related generalizations. Our main goal is to find new structural assumptions on F, that are in principle very far from the Uhlenbeck setting, implying the Hölder continuity for minimizers/solutions. We show that the essential role in our analysis plays the Noether equation, which can be deduced by variations with respect to the internal variable x. Under the so-called splitting condition we show that any minimizer belongs to BMO. Moreover, we show that in case that F is u-independent, then u is Hölder continuous. For F being u dependent, we introduce the so-called one-sided condition and for such F's we show that for p $= 2$ any minimizer is Hölder continuous and for $p \neq 2$, we show that any bounded minimizer is also Hölder continuous. Finally, such results also hold for any (not only bounded) minimizer in case that the principal part of F is strictly convex with respect to the second variable.

Abstracts of short talks

Jan Burczak, Regularity of evolutionary symmetric p-Laplacian. I will present a higher interior regularity result for the symmetric p-Laplace system. The focal point of the talk will be the higher time-regularity proof,based on iterative scheme in Nikolskii-Bochner spaces.

Francesca Crispo, High regularity results of solutions to modified p-Navier-Stokes equations. We consider the following modified p-Navier-Stokes system $-\nabla \cdot (|\nabla u|^{p-2}\nabla u) + (u \cdot \nabla)u + \nabla\pi = f$, $\nabla \cdot u = 0$ in $\mathbb{R}^3$, (1) in the sub-quadratic case $p \in (1, 2)$. This system was considered for the first time in the sixties by Lions. Subsequently the system obtained by replacing ∇u with its symmetric part has been much more studied, due to its connection with the motion of shear-thinning fluids. As this last system is usually called p-Navier-Stokes system, we refer to system (1) as modified p-Navier-Stokes. We find sufficient conditions for the existence of high regular solutions, in the sense of second derivatives in $L^q(\mathbb{R}^3)$, for q varying in $(3,\infty)$. By embedding, we get Hölder continuity of the gradient of solutions. As for the classical Navier-Stokes system in $\mathbb{R}^3$, the uniqueness in the existence class is not achieved.

Martin Kalousek, Homogenization of a non-Newtonian flow through a porous medium. We consider the incompressible viscous non-Newtonian flow through a porous medium. We assume that viscosity is a nonlinear function of the symmetric velocity gradient, i.e. this nonlinear function is a generalization of the power-law case. We provide a mathematical derivation of the law governing a polymer flow through a porous medium using homogenization. The crucial mathematical tool that we use is two-scale convergence, here adopted for Orlicz setting.

Katarzyna Ewa Mazowiecka, Conditional regularity for p-parabolic systems with critical right hand side. We will present the progress in our work on conditional regularity for p-parabolic systems with rhs bounded only by p-th power of gradient of the solution. As of now, we obtained a result assuming that a solution belongs to $L^p(W^{2,p})$ and its BMO norm is uniformly small in time. Our method relies on a Gagliardo-Nirenberg type inequality due to Riviere and Strzelecki. It is joint work with K. Kazaniecki and M. Łasica.

Šárka Nečasová, On the existence of weak solution to the coupled fluid-structure interaction problem for non-Newtonian shear-dependent fluid. We study the existence of weak solution for unsteady fluid-structure interaction problem for shear-thickening flow. The time dependent domain has at one part a flexible elastic wall. The evolution of fluid domain is governed by the generalized string equation with action of the fluid forces. The power law viscosity model is applied to describe shear-dependent non-Newtonian fluids. It is a joint work with A. Hundertmark-Zauskova and M. Lukacova-Medvidova.

Sebastian Schwarzacher, BMO estimates for degenerate parabolic systems. We present results of the non-linear Calderon-Zygmund theory. In the linear stationary case, i.e. Poisson's equation: $\Delta u = \operatorname{div} f$. Here $f \mapsto \nabla u$ is a singular integral operator and therefore a lot of regularity transfers from f to ∇u by the linear Calderon-Zygmund theory. E.g. integrability and Hölder continuity; the borderline case in between integrability and continuity properties is the space of bounded mean oscillations (BMO), which is of special interest. It is the concern of the nonlinear Caldron-Zygmund theory, to derive estimates known for singular integral operators to solutions of non-linear PDE. In this talk, estimates for the p-Laplace are introduced and it is shown how these estimates can be extended to the parabolic p-Laplace and to the p-Stokes equation. Especially the parabolic analog of the borderline BMO case will be discussed.

Lenka Slavíková, Compactness of higher-order Sobolev embeddings. A connection between first-order Sobolev embeddings and isoperimetric inequalities has been known for more than fifty years. In our recent joint paper with Andrea Cianchi and Luboš Pick we have shown that also optimal higher-order Sobolev embeddings are implied by isoperimetric inequalities. In this talk we present a result in the spirit of the previous one, concerning higher-order compact Sobolev embeddings. We study such embeddings on a domain $\Omega \subseteq \mathbb{R}^n$ endowed with a probability measure ν having a Borel density with respect to the Lebesgue measure. We find a condition on a pair of rearrangement-invariant spaces $X(0, 1)$ and $Y(0, 1)$ which suffices to guarantee a compact embedding of the m-th order Sobolev space $V^mX(\Omega, \nu)$ into $Y(\Omega, \nu)$. The condition is given in terms of compactness of certain one-dimensional operator depending on the isoperimetric function of the underlying measure space (Ω, ν). We also show how this result can be applied to characterize higher-order compact Sobolev embeddings on several measure spaces, including John domains, Maz'ya classes of Euclidean domains and product probability spaces, whose standard example is the Gauss space.

Maria Specovius-Neugebauer, Some regularity results for plasticity problems. In this lecture regularity results for the stress velocity for classical variational inequalities modeling elastic plastic problems with hardening are presented. (Joint work with J. Frehse, Bonn)

Paweł Subko, Remark on transport equation with $b \in BV$ and $\mathbf{div}_x b \in BMO$. We investigate the transport equation $\partial_t u(t,x) + b(t,x) \cdot D_x u(t,x) = 0$. Our result improves the classical criteria on uniqueness of weak solutions in the case of irregular coefficients: $b \in BV$, $\mathrm{div}_x b \in BMO$. To obtain the result we use a similar procedure to the DiPerna's and Lions's one developed for Sobolev vector fields. We use renormalization theory for BV vector fields and we employ logarithmic type inequalities to apply energy estimates.

Mikhail Surnachev, On density of smooth functions in weighted Sobolev-Orlicz spaces with variable exponent. In this talk I will discuss the problem of density of smooth functions in weighted Sobolev-Orlicz spaces with variable exponent. The classical result of Meyers and Serrin ("H=W") says that smooth functions are dense in the standard Sobolev spaces. In weighted Sobolev spaces, with a weight from the corresponding Muckenhoupt class, the "H=W" result still holds. An interesting question is how far we can go beyond the Muckenhoupt class setting. I give a sufficient condition for density of smooth functions in weighted Sobolev spaces with variable exponent , which generalizes a recent result by V. Zhikov proved for Sobolev spaces with constant exponent. This work was supported by the Russian Foundation for Basic Research, project no. 14-01-31341.

Jakub Tichý, Higher integrability of generalized Stokes system under perfect slip boundary conditions. We prove an L^q theory result for generalized Stokes system on $\mathcal{C}^{2,1}$ domain complemented with perfect slip boundary conditions and under Φ-growth conditions. Since the interior regularity was obtained in [2], a regularity up to the boundary is our aim. In order to get the main result we use Calderón-Zygmund theory and method developed in [1]. We obtain higher integrability of the first gradient of the solution.

[1] L. A. Caffarelli, I. Peral, I., On $W^{1,p}$ estimates of elliptic equation in divergence form, Comm. Pure and Appl. Math., 51 (1998), 1–21.
[2] L. Diening, P. Kaplický, L^q theory for a generalized Stokes system, Manuscripta Mathematica 141 (2013), no. 1-2, 336–361.

Alexander Ukhlov, Conformal composition operators and Brennan's conjecture. We study composition operators on Sobolev spaces generated by conformal mappings of plane Euclidean domains $\Omega \subset \mathbb{R}^2$ in connection with Brennan's conjecture. Brennan's conjecture states integrability of the complex derivative φ of a plane conformal mapping $\varphi : \Omega \to \mathbb{D}$, $\mathbb{D}$ is the unit disc, in the power $\frac{4}{3} < s < 4$. We prove that Brennan's conjecture holds if and only if φ generates by the composition rule $\varphi*(f) = f \circ \varphi$, $f \in L^1_p(\mathbb{D})$, $2 < p < \infty$, a bounded composition operator

$$\varphi* : L^1_p(\mathbb{D}) \to L^1_q(\Omega), \qquad q = \frac{ps}{p+s-2}.$$

This result has applications in the weighted Sobolev type embedding theory and degenerate elliptic boundary value problems. Joint work with Vladimir Gol'dshtein.

Werner Varnhorn, On local strong solutions of the non homogeneous Navier-Stokes equations. Consider a bounded domain $\Omega \subseteq \mathbb{R}^3$ with smooth boundary $\partial\Omega$, a time interval $[0,T)$, $0 < T \leq \infty$, and in $[0,T) \times \Omega$ the non-homogeneous Navier-Stokes system

$$u_t - \Delta u + u \cdot \nabla u + \nabla p = f,\ u|_{t=0} = u_0,\ \mathrm{div}\, u = k,\ u|_{\partial\Omega} = g,$$

with sufficiently smooth data f, u_0, k, g. In this general case there are mainly known two classes of weak solutions, the class of global weak solutions, similar as in the well known case $k = 0$, $g = 0$, which need not be unique, see [5], and the class of local very weak solutions, see [1], [2], [3], [4], which are uniquely determined, but need neither have differentiability properties nor satisfy the energy inequality. Our aim is to introduce a new class of local strong solutions for the general case $k \neq 0$, $g \neq 0$, satisfying similar regularity and uniqueness properties as in the known case $k = 0$, $g = 0$. For slightly restricted data this class coincides with the corresponding class of very weak solutions yielding new regularity results. Further, through the given data we obtain a control on the interval of existence of the strong solution. Joint work with Reinhard Farwig and Hermann Sohr.

[1] H. Amann, Nonhomogeneous Navier-Stokes equations with integrable low-regularity data, Int. Math. Ser., Kluwer Academic/Plenum Publishing, New York, 2002, 1–26.
[2] H. Amann, Navier-Stokes equations with nonhomogenous Dirichlet data, J. Nonlinear Math. Phys. 10, Suppl. 1 (2003), 1–11.
[3] R. Farwig, G. P. Galdi, H. Sohr, A new class of weak solutions of the Navier-Stokes equations with nonhomogeneous data, J. Math. Fluid Mech. 8 (2006), 423–444.
[4] R. Farwig, H. Kozono, H. Sohr, Very weak, weak and strong solutions to the instationary Navier-Stokes system, Topics on partial differential equations, J. Nečas Center for Mathematical Modeling, Lecture Notes, Vol. 2, P. Kaplický, Š. Nečasová (eds.), pp. 15-68, Prague 2007.
[5] R. Farwig, H. Kozono, H. Sohr, Global weak solutions of the Navier-Stokes equations with nonhomogeneous boundary data and divergence, Rend. Sem. Math. Univ. Padova, 125 (2011), 51–70.