Principal Eigenvalue of Mixed Problem for the Fractional Laplacian: Moving the Boundary Conditions. Applications*

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Abstract

Let Ω be a bounded domain in $\mathbb{R}^N$ and $D, N \subset \mathbb{R}^N \setminus \Omega$ open sets such that $D \cap N = \emptyset$, $\bar{D} \cup \bar{N} = \mathbb{R}^N \setminus \Omega$. We analyze the behavior of the eigenvalues of the following non local mixed problem

$$\begin{cases} (-\Delta)^s u &= \lambda_1(D)\, u & \text{in } \Omega, \\ u &= 0 & \text{in } D, \\ \mathcal{N}_s u &= 0 & \text{in } N, \end{cases}$$

where

$$\mathcal{N}_s u(x) = a_{N,s} \int_\Omega \frac{u(x) - u(y)}{|x-y|^{N+2s}} dy, \quad x \in \mathbb{R}^N \setminus \overline{\Omega}$$

is a natural Neumann boundary condition.

Our first goal is to construct different sequences of problems by modifying the configuration of the sets D and N, and to provide sufficient and necessary conditions on the size and the location of these sets in order to obtain sequences of eigenvalues that in the limit recover the eigenvalues of the Dirichlet or Neumann problem. We will see that the non locality plays a crucial role here, since the sets D and N can have infinite measure, a phenomenon that does not appear in the local case. The second aim of the talk is to study the *mixed Dirichlet-Neumann boundary problem*

$$P_\lambda \equiv \begin{cases} (-\Delta)^s u &= \lambda \dfrac{u}{|x|^{2s}} + u^p & \text{in ,} \\ u &> 0 & \text{in ,} \\ \mathscr{B}_s u &:= u\chi_D + \mathcal{N}_s u \chi_N = 0 & \text{in } \mathbb{R}^N \setminus \Omega, \end{cases}$$

where $N > 2s$, ł > 0 and $0 < p \leq 2^* - 1$, $2^* = \frac{2N}{N-2s}$. We emphasize that the nonlinear term can be critical.

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