Regularity theory for elliptic and parabolic systems and problems in continuum mechanics, 12th – 15th September 2022, Telč

Tuesday, September 13

8:00	Breakfast
8:55	Opening
	Chair: Giuseppe Mingione
9:00	**Domenico Mucci**: Micro-slip-induced multiplicative plasticity: existence of energy minimizers
10:00	Coffee break
	Chair: Emil Wiedemann
10:30	**Lenka Slavíková**: Fractional Orlicz-Sobolev embeddings
11:20	**Emilio Acerbi**: Stable periodic arrangements arising from nonlocal energies
11:55	**Michał Łasica**: Bounds on singularity of minimizers of Rudin-Osher-Fatemi type functionals
12:30	Lunch
15:00	Coffee break
	Chair: Gilles Francfort
15:30	**Michael Bildhauer**: Properties of smooth solutions to the μ-surface equation
16:30	Break
17:00	**Buddhika Priyasad Sembukutti Liyanage**: Maximal L^p-regularity for an abstract evolution equation with applications to closed-loop boundary feedback control problems
17:35	**Daniel Lear Claveras**: Unidirectional flocks in Collective Dynamics
18:10	**Miroslav Bulíček**: Stability of equilibria to generalized Navier–Stokes–Fourier system
19:15	Dinner

Wednesday, September 14

8:00	Breakfast
	Chair: Michael Bildhauer
9:00	**Gilles A. Francfort**: Regularity and rigidity of the stress field in Von Mises plasticity
10:00	Coffee break
10:30	**Yasemin Şengül**: Stability in nonlinear viscoelasticity via monodromy
11:20	**Josef Málek**: On rate-type viscoelastic fluids with stress diffusion and their large-data analysis
12:15	Lunch
15:00	Coffee break
	Chair: Josef Málek
15:30	**Emil Wiedemann**: Weak and measure-valued solutions of the Euler equations
16:30	Break
17:00	**Milan Pokorný**: Continuity equation and vacuum regions in compressible flows
17:35	**Anna Abbatiello**: The Oberbeck-Boussinesq system with non-local boundary conditions
18:10	**Pablo Alexei Gazca Orozco**: Numerical approximation of fluids with semismooth implicitly constituted laws
19:15	Dinner

Thursday, September 15

8:00	Breakfast
	Chair: Domenico Mucci
9:00	**Giuseppe Mingione**: Hopf, Caccioppoli and Schauder, reloaded
10:00	Coffee break
	Chair: Emilio Acerbi
10:30	**Cristiana De Filippis**: Quasiconvexity meets nonlinear potential theory
11:20	**Anna Kh.Balci**: Variational problems and Lavrentiev gap in partial Sobolev spaces of differential forms
11:55	Closing
12:00	Lunch
13:00	Departure

Abstracts of talks

Anna Abbatiello: Regularity of solution to flow of non-Newtonian fluids with concentration dependent power-law index. We consider the Oberbeck-Boussinesq system with non-local boundary conditions arising as a singular limit of the full Navier-Stokes-Fourier system in the regime of low Mach and low Froude number. The existence of strong solutions is shown on a maximal time interval $[0, T_{\max})$. Moreover, $T_{\max} = \infty$ in the two dimensional setting. This is a joint work with Eduard Feireisl.

Emilio Acerbi: Stable periodic arrangements arising from nonlocal energies. Competition between phases may give rise to interesting periodic local minimizers. To identify the candidates among stationary points one has to find an analogous of the positive–second–derivative criterion. In some cases linked with copolymers or reaction diffusion, it is possible to prove that some periodic stationary points are local minimizers, and in the case of lamellar solutions to give necessary and sufficient conditions for local minimality.

Michael Bildhauer: Properties of smooth solutions to the μ-surface equation. We are interested in a class of equations related to the minimal surface equation but not necessarily governed by the geometric properties of minimal surfaces. More precisely we consider smooth solutions the equation

$$\operatorname{div}\left[\frac{g'\big(|\nabla u|\big)}{|\nabla u|}\nabla u\right] = 0\,,$$

where the function g satisfies a linear growth condition together with some additional reasonable assumptions. In the first part of the talk the situation in some sense is not too far away from the minimal surface case and we recover particular geometric properties known from the minimal surface case. In the second part a theorem of Bers and Finn on the removability of isolated singularities is discussed. As the main tool we study the comparison properties of generalized catenoids. We finally sketch some recent developments.

Miroslav Bulíček: Stability of equilibria to generalized Navier–Stokes–Fourier system. We consider a generalized Newtonian incompressible heat conducting fluid with prescribed nonuniform temperature on the boundary and with the no-slip boundary conditions for the velocity. The fluid occupies a three dimensional domain. No external body forces are applied to the fluid. For the constitutively determined part of the Cauchy stress growing sufficiently fast (with power law index bigger than 11/5), we identify a class of proper solutions converging to the equilibria exponentially in a suitable metric. Consequently, the equilibrium is nonlinearly stable and attracts all weak solutions from that class. The proper solutions exist and satisfy entropy equality.

Cristiana De Filippis: Quasiconvexity meets nonlinear potential theory. A classical problem in the regularity theory for vector-valued minimizers of multiple integrals consists in proving their smoothness outside a negligible set, cf. Evans (*ARMA* '86), Acerbi & Fusco (*ARMA* '87), Duzaar & Mingione (*Ann. IHP-AN* '04), Schmidt (*ARMA* '09). In this talk, I will show how to infer sharp partial regularity results for relaxed minimizers of degenerate/singular, nonuniformly elliptic quasiconvex functionals, using tools from nonlinear potential theory. In particular, in the setting of functionals with (p, q)-growth - according to the terminology introduced by Marcellini (*Ann. IHP-AN* '86; *ARMA* '89) - I will derive optimal local regularity criteria under minimal assumptions on the data. This talk is partly based on joint work with Bianca Stroffolini (University of Naples Federico II).

Gilles A. Francfort: Regularity and rigidity of the stress field in Von Mises plasticity. In this joint work with J. F. Babadjian, inspired from prior work with A. Giacomini and J. J. Marigo, we start an investigation of spatial hyperbolicity in Von Mises elasto-plasticity, the ultimate goal being an adjudication of the uniqueness of the plastic strain. After discussing a specific example where uniqueness, or lack thereof, can be established, I will present partial results, focussing on a 2d simplified model.

Pablo Alexei Gazca Orozco: Numerical approximation of fluids with semismooth implicitly constituted laws. We propose a semismooth Newton method for non-Newtonian models of incompressible flow where the constitutive relation between the shear stress and the symmetric velocity gradient is given implicitly; as a motivating example we consider the Bingham model for viscoplastic flow. The proposed method avoids the use of variational inequalities and is based on a particularly simple regularisation introduced recently by Bulíček et al., for which the (weak) convergence of the approximate stresses is known to hold. The system is analysed at the function space level and results in mesh-independent behaviour of the nonlinear iterations.

Anna Kh.Balci: Variational problems and Lavrentiev gap in partial Sobolev spaces of differential forms. We study variational problems in generalised Sobolev-Orlicz spaces of differential forms. In particular we provide results on density of smooth functions and design examples on Lavrentiev gap for partial spaces of differential forms. The construction is based on a Cantor type “singular set”. As the application we demonstrate the Lavrentiev for several models including borderline case of double-phase potential. The talk based on join work with Mikhail Surnachev.

Michał Łasica: Bounds on singularity of minimizers of Rudin–Osher–Fatemi type functionals. Coercive convex functionals of linear growth in the gradient of the argument generically attain their minima in the space of functions of bounded variation, i.e. the gradient of the minimizer is a vector measure. One class of such functionals are Rudin-Osher-Fatemi (ROF) type models known for their applications to image processing. ROF-type functionals consist of a term of linear growth and a fidelity term measuring the distance between the argument and a given function (“noisy image”). In this talk, we will discuss recent results on control of the Lebesgue-singular part of the gradient of the minimizer in terms of the datum.

Daniel Lear Claveras: Unidirectional flocks in Collective Dynamics. In this talk we will focus on the so-called Cucker-Smale model, which encode one of the simplest communication protocols that lead to emergence of two fundamental phenomena of collective action: alignment and flocking. Such systems arise in a variety of applications including biological, social and technological contexts. Kinetic and hydrodynamic models will be presented and the problems of global well-possendess, long time behavior, and stability of flocks on the macroscopic level will be addressed. Finally, we discuss unidirectional flocks, which have been recently introduced and used to obtain some results in higher dimensions.

Josef Málek: On rate-type viscoelastic fluids with stress diffusion and their large-data analysis. This presentation is based on joint works with Michal Bathory (University of Vienna, Austria), Miroslav Bulíček (Charles University, Prague) and Casey Rodrigues (University of North Carolina, USA).

We present the result concerning the large-data and long-time existence of a weak solution to an initial- and boundary-value problem associated with a PDE system governing unsteady flows of a robust class of rate-type viscoelastic fluid with stress diffusion in two and three dimensions. The fluid is described by the incompressible Navier-Stokes equations for the velocity $\mathbf{v}$, coupled with a diffusive variant of a combination of the Oldroyd-B and the Giesekus models for a tensor $\mathbb{B}$. By a proper choice of the constitutive relations for the Helmholtz free energy (which, however, is non-standard in the current literature, despite the fact that this choice is well motivated from the point of view of physics) and for the energy dissipation, we are able to proves that $\mathbb{B}$ enjoys the same regularity as $\mathbf{v}$ in the classical three-dimensional Navier-Stokes equations. This enables us to handle any kind of objective derivative of $\mathbb{B}$, thus obtaining existence results for the class of diffusive Johnson-Segalman models as well. Moreover, using a suitable approximation scheme, we are able to show that $\mathbb{B}$ remains positive definite if the initial datum was a positive definite matrix (in a pointwise sense). This talk is based on the results published in [1] and [2].

[1] Bathory, Michal and Bulíček, Miroslav and Málek, Josef *Large data existence theory for three-dimensional unsteady flows of rate-type viscoelastic fluids with stress diffusion.* Adv. Nonlinear Anal. **10** (2021) 501–521.

[2] Bulíček, Miroslav and Málek, Josef and Rodriguez, Casey *Global well-posedness for two-dimensional flows of viscoelastic rate-type fluids with stress diffusion.* J. Math. Fluid Mech. **24** (2022) Paper No. 61.

Giuseppe Mingione: Hopf, Caccioppoli and Schauder, reloaded. So called Schauder estimates are in fact a contribution, at various stages, of Hopf, Caccioppoli and Schauder, between the end of the 20s and the beginning of the 30s. Later on, they were extended, with various degrees of precision, to nonlinear uniformly elliptic equations. I will present the solution to the longstanding open problems of proving estimates of such kind in the nonuniformly elliptic case and for minima of non-differentiable functionals (again considered in the nonuniformly elliptic case). From joint work with Cristiana De Filippis.

Domenico Mucci: Micro–slip–induced multiplicative plasticity: existence of energy minimizers. To account for material slips at microscopic scale, we take deformation mappings as SBV functions φ, which are orientation-preserving outside a jump set taken to be two-dimensional and rectifiable. For their distributional derivative $F = D\varphi$ we consider the common multiplicative decomposition $F = F^e F^p$ into so-called elastic and plastic factors, the latter a measure. Then, we consider a polyconvex energy with respect to F^e, augmented by the measure $|\mathrm{curl} F^p|$. For this type of energy we prove existence of minimizers in the space of SBV maps. We avoid self-penetration of matter. Our analysis rests on a representation of the slip system in terms of currents (in the sense of geometric measure theory) with both $\mathbb{Z}^3$ and $\mathbb{R}^3$ valued multiplicity. The two choices make sense at different spatial scales. They describe separate but not alternative modeling choices. The first one is particularly significant for periodic crystalline materials at a lattice level. The latter covers a more general setting and requires to account for an energy extra term involving the slip boundary size. We include a generalized (and weak) tangency condition; the resulting setting agrees to gliding and cross-slip mechanisms. The possible highly articulate structure of the jump set allows one to consider the resulting setting even as an approximation of climbing mechanisms. This is a joint work with P. M. Mariano (DICEA, University of Firenze, Italy).

Milan Pokorný: Continuity equation and vacuum regions in compressible flows. We investigate the creation and properties of eventual vacuum regions in the weak solutions of the continuity equation, in general, and in the weak solutions of compressible Navier–Stokes equations, in particular. The main results are based on the analysis of renormalized solutions to the continuity and pure transport equations and their inter-relations. The presentation is based on the paper Novotný, Pokorný: Continuity equation and vacuum regions in compressible flows, J. Evol. Equ. 21 (2021), no. 3, 2891–2922.

Buddhika Priyasad Sembukutti Liyanage: Maximal L^p-regularity for an abstract evolution equation with applications to closed-loop boundary feedback control problems. We present an abstract maximal L^p-regularity result up to $T = \infty$, that is tuned to capture (linear) Partial Differential Equations of parabolic type, defined on a bounded domain and subject to finite dimensional, stabilizing, feedback controls acting on (a portion of) the boundary. Illustrations include, beside a more classical boundary parabolic example, two more recent settings: the 3d-Navier-Stokes equations with finite dimensional, localized, boundary tangential feedback stabilizing controls as well as Boussinesq systems with finite dimensional, localized, feedback, stabilizing, Dirichlet boundary control for the thermal equation.

Yasemin Şengül: Stability in nonlinear viscoelasticity via monodromy. We consider nonlinear viscoelasticity of strain-rate type under some assumptions allowing for solid phase transformations. We consider the dynamical problem as an elliptic regularisation of the quasistatic case and formulate the latter as a gradient flow in one space dimension, leading to existence and uniqueness of solutions. By approximating general initial data by those in which the deformation gradient takes only finitely many values, we show that under suitable hypotheses on the stored-energy function the deformation gradient is instantaneously bounded and bounded away from zero, which is an important feature for the stability analysis. In order to prove convergence as time tends to infinity of solutions to a single equilibrium, it seems necessary to impose a nondegeneracy condition on the constitutive equation for the stress. We will investigate this condition and show how in some cases it can be proved using the monodromy group of a holomorphic function.

Lenka Slavíková: Fractional Orlicz-Sobolev embeddings. We establish the optimal Orlicz target space for embeddings of fractional-order Orlicz-Sobolev spaces in the Euclidean space. We also present an improved embedding with an Orlicz-Lorentz target space, which is optimal in the broader class of all rearrangement-invariant spaces. We will consider both spaces of order less than one as well as higher-order spaces. This is a joint work with A. Alberico, A. Cianchi and L. Pick.

Emil Wiedemann: Weak and measure–valued solutions of the Euler equations. Several notions of weak or "very weak" solutions have been suggested for the incompressible and compressible Euler systems, motivated by the lack of a satisfactory well-posedness theory for these equations in turbulent regimes. Surprisingly, the speaker and L. Székelyhidi showed in 2012 that distributional and measure-valued solutions of the incompressible system are in a sense the same, although the latter had been expected to be a much weaker notion. In this talk, we turn to the isentropic compressible Euler system, where the situation is fundamentally different. Our approach borrows and develops several tools from the theory of multiple integrals, such as quasiconvexity, Ball-James rigidity, or Müller-Zhang truncation. Joint work with D. Gallenmüller.