The Two-way Error Component Regression Model

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Highlights

Introduction

Fixed effects

Random effects

Maximum likelihood estimation

Prediction

Example(s)

Two-way Error Component

$$y_{it} = \alpha + \mathbf{X}'_{it}\beta + u_{it} \tag{1}$$

- $i = 1, \ldots, N$, individual, cross-section,
- $t = 1, \ldots, T$, time-series
- K explanatory variables

$$u_{it} = \mu_i + \lambda_t + \nu_{it} \tag{2}$$

- μ_i, individual effect $\sum_{i=1}^{N} \mu_i = 0$
- λ_t time effect not included in the regression:
  - strike year
  - oil embargo
  - laws restricting smoking

$\sum_{t=1}^{T} \lambda_t = 0$

Wallace and Hussain (1969), Nerlove (1971) and Amemiya (1971)

Vector form

$$\mathbf{u} = Z_\mu \mu + Z_\lambda \lambda + \nu \tag{3}$$

- $\mathbf{u}' = (u_{11}, \ldots, u_{1T}, \ldots, u_{NT})$
- $Z_\mu = I_N \otimes \iota_T$ matrix of individual dummies
- $Z_\lambda = \iota_N \otimes I_T$ matrix of time dummies, dim $NT \times T$
  - ι_N vector of ones, dim N
  - I_T identity matrix, dim T
- $\nu' = (\nu_{11}, \ldots, \nu_{1T}, \ldots, \nu_{NT})$

- $Z_\lambda Z_\lambda' = J_N \otimes I_T$
  - J_N matrix of ones, dim N
- $Z_\lambda (Z_\lambda' Z_\lambda)^{-1} Z_\lambda' = \bar{J}_N \otimes I_T$ projection matrix
  - $\bar{J}_N = J_N / N$
- $(\bar{J}_N \otimes I_T) y$ predicted values: $\bar{y}_{.t} = \sum_{i=1}^{N} y_{it} / N$

Fixed effects

- μ_i and λ_t fixed parameters
- $\nu_{it} \sim IID(0, \sigma_\nu^2)$
- $\mathbf{X}_{it}$ independent of ν_{it} for all i and t

regressing the original problem (1) yields several troubles

- enormous loss in df due to $\{(N-1)+(T-1)\}$ dummies
- blurring multicollinearity among regressors
- inverting large $(N+T+K-1)$ matrix

to obtain the fixed effect estimates of β serves Within transformation given by (Wallace, Hussain: 1969):

$$\begin{aligned} Q &= E_N \otimes E_T = (I_N - \bar{J}_N) \otimes (I_T - \bar{J}_T) = \\ &= I_N \otimes I_T - I_N \otimes \bar{J}_T - \bar{J}_N \otimes I_T + \bar{J}_N \otimes \bar{J}_T \end{aligned} \tag{4}$$

Within estimator

- $\tilde{\mathbf{y}} = Q\mathbf{y}$ has elements $\tilde{y}_{it} = (y_{it} - \bar{y}_{i.} - \bar{y}_{.t} + \bar{y}_{..})$
- regression of $\tilde{y}$ on $\tilde{X} = QX$ yields the *Within* estimator

$$\tilde{\beta} = (X'QX)^{-1}X'Qy \tag{5}$$

- $\tilde{\alpha} = \bar{y}_{..} - \tilde{\beta}\ \bar{x}_{..}$
- $\tilde{\mu}_i = (\bar{y}_{i.} + \bar{y}_{..}) - \tilde{\beta}(\bar{x}_{i.} + \bar{x}_{..})$
- $\tilde{\lambda}_t = (\bar{y}_{.t} + \bar{y}_{..}) - \tilde{\beta}(\bar{x}_{.t} + \bar{x}_{..})$

- Within estimator can not estimate the effect of time- and individual-invariant variables since Q transformation wipes out these variables
- if estimating (5) with the standard regression package to get the proper variance-covariance matrix one needs to divide the estimated one by $(NT - N - T + 1 - K)$ and multiply by $(NT - K)$

Testing for fixed effects

- joint significance of the dummy variables

$$H_0 : \mu_i = 0 \quad \text{and} \quad \lambda_t = 0$$

  - RRSS from pooled OLS: $y_{it} = \alpha + \mathbf{X}'_{it}\beta + \nu_{it}$
  - URSS from Within regression

$$F_1 = \frac{(\text{RRSS} - \text{URSS})/(N+T-2)}{\text{URSS}/(NT-N-T+1-K)} \sim F_{(N+T-2),(NT-N-T+1-K)}$$

- existence of individual effects: $H_2 : \mu_i = 0$
  - RRSS from regression with time dummies only

$$y_{it} - \bar{y}_{.t} = (x_{it} - \bar{x}_{.t})\beta + (u_{it} - \bar{u}_{.t})$$

$$F_2 \sim F_{(N-1),(NT-N-T+1-K)}$$

Random effects

$$\mu_i \sim IID(0,\sigma^2_\mu) \quad \lambda_t \sim IID(0,\sigma^2_\lambda) \quad \nu_{it} \sim IID(0,\sigma^2_\nu)$$

- mutualy independent as well as with $\mathbf{X}_{it}$
- inference for random sample from large population

$$\begin{aligned}\Omega &= E(uu') = Z_\mu E(\mu\mu')Z'_\mu + Z_\lambda E(\lambda\lambda')Z'_\lambda + \sigma^2_\nu I_{NT} \\ &= \sigma^2_\mu(I_N \otimes J_T) + \sigma^2_\lambda(J_N \otimes I_T) + \sigma^2_\nu(I_N \otimes I_T) \qquad (6)\end{aligned}$$

$$cov(u_{it},u_{js}) = \begin{cases} \sigma^2_\mu + \sigma^2_\lambda + \sigma^2_\nu & i=j, s=t \\ \sigma^2_\mu & i=j, s\neq t \\ \sigma^2_\lambda & i\neq j, s=t \\ 0 & i\neq j, s\neq t \end{cases} \qquad (7)$$

$$\Omega = \sigma^2_\mu \; T((E_N + \bar{J}_N)\otimes\bar{J}_T) + \sigma^2_\lambda \; N(\bar{J}_N\otimes(E_T+\bar{J}_T)) + \sigma^2_\nu((E_N+\bar{J}_N)\otimes(E_T+\bar{J}_T))$$

Matrix spectral decomposition

$$\Omega = \sum_{i=1}^{4} \lambda_i Q_i \tag{8}$$

i	λ_i	Q_i	$\# \lambda_i$
1	σ_ν^2	$E_N \otimes E_T$	$NT - N - T + 1$
2	$T\sigma_\mu^2 + \sigma_\nu^2$	$E_N \otimes \bar{J}_T$	$N - 1$
3	$N\sigma_\lambda^2 + \sigma_\nu^2$	$\bar{J}_N \otimes E_T$	$T - 1$
4	$T\sigma_\mu^2 + N\sigma_\lambda^2 + \sigma_\nu^2$	$\bar{J}_N \otimes \bar{J}_T$	1

- λ_i are the characteristic roots of Ω
- Q_i are the matrices of eigenprojectors, symetric and idempotent, pairwise orthogonal and $\sum_{i=1}^{4} Q_i = I_{NT}$

$$\Omega^r = \sum_{i=1}^{4} \lambda_i^r Q_i \tag{9}$$

Transformation

$$\sigma_\nu \Omega^{-1/2} = \sum_{i=i}^{4} (\sigma_\nu / \lambda_i^{1/2}) Q_i \tag{10}$$

element of $y^* = \sigma_\nu \Omega^{-1/2} y$ is given by

$$y^*_{it} = y_{it} - \theta_1 \bar{y}_{i.} - \theta_2 \bar{y}_{.t} + \theta_3 \bar{y}_{..} \tag{11}$$

- $\theta_1 = 1 - (\sigma_\nu / \lambda_2^{1/2})$
- $\theta_2 = 1 - (\sigma_\nu / \lambda_3^{1/2})$
- $\theta_3 = \theta_1 + \theta_2 + (\sigma_\nu / \lambda_4^{1/2}) - 1$

GLS estimator of (1) can be obtained as OLS of y^* on $Z^* = \sigma_\nu \Omega^{-1/2} Z$ (Fuller, Battese: 1974)

Best quadratic unbiased estimators

from $Q_i\mathbf{u} \sim (0, \lambda_i Q_i)$ follows that $\hat{\lambda}_i = \mathbf{u}' Q_i \mathbf{u} / tr(Q_i)$

- is the BQU estimator for $i = 1, 2, 3$
- estimators are minimum variance unbiased under normality of the disturbances (Graybill: 1961)
- estimates of variance by OLS residuals (Wallace, Hussain: 1696) are unbiased and consistent, but inefficient
- Within residuals has the same asymptotic distribution as the true disturbances (Amemiya: 1971):

$$\begin{pmatrix} \sqrt{NT}(\hat{\sigma}_\nu^2 - \sigma_\nu^2) \\ \sqrt{N}(\hat{\sigma}_\mu^2 - \sigma_\mu^2) \\ \sqrt{T}(\hat{\sigma}_\lambda^2 - \sigma_\lambda^2) \end{pmatrix} \sim N\left(0, \begin{pmatrix} 2\sigma_\nu^4 & 0 & 0 \\ 0 & 2\sigma_\mu^4 & 0 \\ 0 & 0 & 2\sigma_\lambda^4 \end{pmatrix}\right) \qquad (12)$$

- both OLS and Within residuals introduces bias into estimates. the correction of df depend upon traces given by authors

Three LS regressions

to estimate the variance components from corresponding MSE of the regressions (Swamy, Arora: 1972)

$$\hat{\hat{\lambda}}_i = [y'Q_i y - y'Q_i X(X'Q_i X)^{-1}X'Q_i y]/df_i \qquad (13)$$

- Within regression for $\sigma_\nu^2 = \lambda_1$, $df_1 = NT - N - T + 1 - K$
- Between individuals regression of $(\bar{y}_{i.} - \bar{y}_{..})$ on $(\bar{X}_{i.} - \bar{X}_{..})$ for $\sigma_\mu^2 = (\lambda_2 - \sigma_\nu^2)/T$, $df_2 = N - 1 - K$
- Between time-periods regression of $(\bar{y}_{.t} - \bar{y}_{..})$ on $(\bar{X}_{.t} - \bar{X}_{..})$ for $\sigma_\lambda^2 = (\lambda_3 - \sigma_\nu^2)/T$, $df_3 = T - 1 - K$

Stacked regressions

$$\begin{pmatrix} Q_1 y \\ Q_2 y \\ Q_3 y \end{pmatrix} = \begin{pmatrix} Q_1 X \\ Q_2 X \\ Q_3 X \end{pmatrix} \beta + \begin{pmatrix} Q_1 u \\ Q_2 u \\ Q_3 u \end{pmatrix} \tag{14}$$

since $Q_i \iota_{NT} = 0$ and the transformed error has mean 0 and covariance matrix $diag[\lambda_i Q_i]$ for $i = 1, 2, 3$

$$\hat{\beta}_{GLS} = W_1 \tilde{\beta}_W + W_2 \hat{\beta}_B + W_3 \hat{\beta}_C \tag{15}$$

matrix weighted average of Within, Between individuals and Between time-periods estimator

- if $\sigma_\mu^2 = \sigma_\lambda^2 = 0$, $\hat{\beta}_{GLS}$ reduces to $\hat{\beta}_{OLS}$
- as T and $N \to \infty$, λ_2 and $\lambda_3 \to \infty$, $\hat{\beta}_{GLS}$ tends to $\tilde{\beta}_W$

Maximum likelihood estimation

normality needed for errors

$$\log L \propto -1/2 \log |\Omega| - 1/2(y - Z\gamma)'\Omega^{-1}(y - Z\gamma) \qquad (16)$$

$$\begin{aligned}
\frac{\partial \log L}{\partial \gamma} &= Z'\Omega^{-1}y - (Z'\Omega^{-1}Z)\gamma = 0 \qquad (17) \\
\frac{\partial \log L}{\partial \sigma_\nu^2} &= -1/2tr\Omega^{-1} + 1/2u'\Omega^{-2}u = 0 \\
\frac{\partial \log L}{\partial \sigma_\mu^2} &= -1/2tr\Omega^{-1}(I_N \otimes J_T) + 1/2u'\Omega^{-2}(I_N \otimes J_T)u = 0 \\
\frac{\partial \log L}{\partial \sigma_\lambda^2} &= -1/2tr\Omega^{-1}(J_N \otimes I_T) + 1/2u'\Omega^{-2}(J_N \otimes I_T)u = 0
\end{aligned}$$

iterative solution needed for normal equations (Amemiya: 1971).
estimates of variance consistent with asymptotic distribution (12)

Concentrated likelihood

from $|\Omega|^{-1} = (\sigma_\nu^2)^{-NT}(\phi_2^2)^{N-1}(\phi_3^2)^{T-1}\phi_4^2$

$$LL(\alpha,\beta,\sigma_\nu^2,\phi_2^2,\phi_3^2) \propto -(NT/2)\log\sigma_\nu^2 + (N/2)\log\phi_2^2 + \quad (18)$$
$$+(T/2)\log\phi_3^2 - (1/2)\log[\phi_2^2+\phi_3^2-\phi_2^2\phi_3^2] - 1/(2\sigma_\nu^2)u'\Sigma^{-1}u$$

where $\phi_i = \sigma_\nu^2/\lambda_i$ and $\Sigma = \Omega/\sigma_\nu^2$

define $d = y - X\beta$, $u = d - \iota_{NT}\alpha$ follows
$\hat{\alpha} = \iota'_{NT}d/NT$ and $\hat{\sigma}_\nu^2 = u'\Sigma^{-1}u/NT$
$u = d - \iota_{NT}\hat{\alpha} = (I_{NT} - \bar{J}_{NT})d$

$$(I_{NT}-\bar{J}_{NT})\Sigma^{-1}(I_{NT}-\bar{J}_{NT}) = Q_1 + \phi_2^2 Q_2 + \phi_3^2 Q_3$$

$$LL_C(\beta,\phi_2^2,\phi_3^2) \propto -(NT/2)\log[d'(Q_1+\phi_2^2Q_2+\phi_3^2Q_3)d] + \quad (19)$$
$$+(N/2)\log\phi_2^2 + (T/2)\log\phi_3^2 - (1/2)\log[\phi_2^2+\phi_3^2-\phi_2^2\phi_3^2]$$

Maximizing iteration

maximizing LL_C over β, given ϕ_2^2 and ϕ_3^2

$$\hat{\beta} = [X'(Q_1 + \phi_2^2 Q_2 + \phi_3^2 Q_3)X]^{-1} X'(Q_1 + \phi_2^2 Q_2 + \phi_3^2 Q_3)y \quad (20)$$

maximizing LL_C over ϕ_2^2, given β and ϕ_3^2

$$\frac{\partial \log L}{\partial \sigma_\lambda^2} = -\frac{NT}{2} \frac{d' Q_2 d}{d'[Q_1 + \phi_2^2 Q_2 + \phi_3^2 Q_3] d} + \frac{N}{2} \frac{1}{\phi_2^2} - \frac{1}{2} \frac{(1 - \phi_3^2)}{[\phi_2^2 + \phi_3^2 - \phi_2^2 \phi_3^2]} \quad (21)$$

can be expressed as

$$a\phi_2^4 + b\phi_2^2 + c = 0 \quad (22)$$

$a = -[NT - N + 1](1 - \phi_3^2)(d' Q_2 d)$
$b = (1 - \phi_3^2)(N - 1)d'[Q_1 + \phi_3^2 Q_3]d - \phi_3^2 (T - 1)N(d' Q_2 d)$
$c = N\ \phi_3^2\ d'[Q_1 + \phi_3^2 Q_3]d$

Iteration properties

for fixed $0 < \phi_3^2 < 1$

- if $\phi_2^2 = 0$, (20) $\hat{\beta}_{BW} = [X'(Q_1 + \phi_3^2 Q_3)X]^{-1} X'(Q_1 + \phi_3^2 Q_3)y$
    - $\hat{\beta}_W = [X'Q_1X]^{-1}X'Q_1y$ Within
    - $\hat{\beta}_C = [X'Q_3X]^{-1}X'Q_3y$ Between time
- if $\phi_2^2 \to \infty$, (20) $\hat{\beta}_B = [X'Q_2X]^{-1}X'Q_2y$ Between individual

$$\hat{\phi}_2^2 = \left[-b - \sqrt{b^2 - 4ac}\right] / (2a) \tag{23}$$

is the unique positive root since $a < 0$ and $c > 0$ (Baltagi, Li: 1992)
and since ϕ_3^2 is fixed using $\hat{Q}_1 = Q_1 + \phi_3^2 Q_3$:

$$\hat{\beta} = [X'(\hat{Q}_1 + \phi_2^2 Q_2)X]^{-1} X'(\hat{Q}_1 + \phi_2^2 Q_2)y \tag{24}$$

- if started from $\hat{\beta}_{BW}$, sequence $\phi_2^2(it)$ is strictly increasing
- if started from $\hat{\beta}_B$, sequence $\phi_2^2(it)$ is strictly decreasing

Best linear unbiased predictor

$$u_{i,T+S} = \mu_i + \lambda_{T+S} + \nu_{i,T+S} \tag{25}$$

$$E(u_{i,T+S}, u_{jt}) = \sigma_\mu^2 \quad i = j \tag{26}$$

$w = E(u_{i,T+S} u) = \sigma_\mu^2 (l_i \otimes \iota_T)$
where l_i is to ith column of I_N

$$w'\Omega^{-1} = \sigma_\mu^2 (l_i' \otimes \iota_T') \left[\sum_{i=1}^{4} \frac{1}{\lambda_i} Q_i \right] \tag{27}$$

$$= \frac{\sigma_\mu^2}{\lambda_2} \left[(l_i' \otimes \iota_T') - \iota_{NT}'/N \right] + \frac{\sigma_\mu^2}{\lambda_4} (\iota_{NT}'/N) \tag{28}$$

$(l_i' \otimes \iota_T') Q_1 = 0 \quad (l_i' \otimes \iota_T') Q_2 = (l_i' \otimes \iota_T') - \iota_{NT}'/N$
$(l_i' \otimes \iota_T') Q_3 = 0 \quad (l_i' \otimes \iota_T') Q_4 = \iota_{NT}'/N$

typical element of $w'\Omega^{-1}\hat{u}_{GLS}$ where $\hat{u}_{GLS} = y - Z\hat{\delta}_{GLS}$ is

$$\frac{T\sigma_\mu^2}{T\sigma_\mu^2 + \sigma_\nu^2}(\bar{\hat{u}}_{i.,GLS} - \bar{\hat{u}}_{..,GLS}) + \frac{T\sigma_\mu^2}{T\sigma_\mu^2 + N\sigma_\lambda^2 + \sigma_\nu^2}\bar{\hat{u}}_{..,GLS} \qquad (29)$$

$$\text{or} \quad \frac{T\sigma_\mu^2}{T\sigma_\mu^2 + \sigma_\nu^2}\bar{\hat{u}}_{i.,GLS} + T\sigma_\mu^2\left[\frac{1}{\lambda_4} - \frac{1}{\lambda_2}\right]\bar{\hat{u}}_{..,GLS}$$

if Z contains a constant, than $\iota'_{NT}\Omega^{-1}\hat{u}_{GLS} = 0$ and using $\iota'_{NT}\Omega^{-1} = \iota'_{NT}/\lambda_4$ one gets $\bar{\hat{u}}_{..,GLS} = 0$ and the BLUP corrects the GLS prediction by a fraction of the mean of GLS residuals of ith individual

$$\hat{y}_{i,T+S} = Z_{i,T+S}\hat{\delta}_{GLS} + \left(\frac{T\sigma_\mu^2}{T\sigma_\mu^2 + \sigma_\nu^2}\right)\bar{\hat{u}}_{i.,GLS} \qquad (30)$$

Software

- LIMDEP, RATS, SAS, TSP and GAUSS (Blanchard: 1996)
- SAS, Stata for larga data sets
- OX, GAUSS for programming estimation and tests
- LIMDEP, TSP, EViews or Stata for simple panel data
- R packages:
  - **plm** for LS procedures, contains example datasets
  - **splm** for MLE procedure (for one way error only)

Two-way error estimation methods

- OLS
- Within estimator
- Swamy and Arora (1972)
- Wallace and Hussain (1969)
- Amemiya (1971)

negative variance estimates indicates zero or small value

Public capital productivity

Cobb-Douglas production function

$$\ln Y = \alpha + \beta_1 \ln K_1 + \beta_2 \ln K_2 + \beta_3 \ln l + \beta_4 U + u \qquad (31)$$

- Y, gross state product, *gsp*
- K_1, public capital, *pcap*
- K_1, private capital, *pc*
- L, labour input as payrolls, *emp*
- U, unemployment rate, *unemp*

annual observations for 48 states over 1970-86 (Munnell: 1990)
dataset Produc.prn on the Wiley web site

	log.pcap.	log.pc.	log.emp.	unemp	idios	id	time
ols	0.155	0.309	0.594	-0.007			
wit	-0.030	0.169	0.769	-0.004			
bln	0.179	0.302	0.576	-0.004			
bTP	0.132	1.192	-0.276	-0.032			
swA	0.018	0.266	0.745	-0.005	0.034	0.083	0.010
waH	0.028	0.260	0.738	-0.005	0.036	0.079	0.016
ame	0.002	0.216	0.770	-0.004	0.034	0.156	0.026

Table: Estimated coefficients from Produc data

	log.pcap.	log.pc.	log.emp.	unemp
ols	0.017	0.010	0.014	0.001
wit	0.027	0.028	0.028	0.001
bln	0.072	0.042	0.056	0.010
bTP	0.334	0.224	0.374	0.008
swA	0.023	0.021	0.024	0.001
waH	0.023	0.021	0.023	0.001
ame	0.025	0.024	0.026	0.001

Table: Estimated std. errors from Produc data

```
Twoways effects Pooling Model

Call:
plm(formula = log(gsp) ~ log(pcap) + log(pc) + log(emp) + unemp,
    data = Produc, effect = "twoways", model = "pooling",
        index = c("state", "year"))

Balanced Panel: n=48, T=17, N=816

Residuals :
     Min.    1st Qu.     Median    3rd Qu.       Max.
-0.232000 -0.061000 -0.000102  0.050900  0.351000

Coefficients :
               Estimate Std. Error t-value  Pr(>|t|)
(Intercept)  1.6433023  0.0575873 28.5359 < 2.2e-16 ***
log(pcap)    0.1550070  0.0171538  9.0363 < 2.2e-16 ***
log(pc)      0.3091902  0.0102720 30.1003 < 2.2e-16 ***
log(emp)     0.5939349  0.0137475 43.2032 < 2.2e-16 ***
unemp       -0.0067330  0.0014164 -4.7537 2.363e-06 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Total Sum of Squares:    849.81
Residual Sum of Squares: 6.2942
R-Squared      :  0.99259
      Adj. R-Squared :  0.98651
F-statistic: 27171.7 on 4 and 811 DF, p-value: < 2.22e-16
```

```
Twoways effects Within Model

Call:
plm(formula = log(gsp) ~ log(pcap) + log(pc) + log(emp) + unemp,
    data = Produc, effect = "twoways", model = "within",
        index = c("state", "year"))

Balanced Panel: n=48, T=17, N=816

Residuals :
     Min.    1st Qu.    Median   3rd Qu.      Max.
-0.160000 -0.018000 -0.000859  0.016700  0.171000

Coefficients :
             Estimate Std. Error t-value  Pr(>|t|)
log(pcap) -0.0301761  0.0269365 -1.1203 0.2629606
log(pc)    0.1688280  0.0276563  6.1045 1.655e-09 ***
log(emp)   0.7693062  0.0281418 27.3368 < 2.2e-16 ***
unemp     -0.0042211  0.0011388 -3.7065 0.0002257 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Total Sum of Squares:    3.5889
Residual Sum of Squares: 0.87944
R-Squared       :  0.75496
      Adj. R-Squared :  0.69204
F-statistic: 576.134 on 4 and 748 DF, p-value: < 2.22e-16
```

```
Twoways effects Random Effect Model
   (Swamy-Arora's transformation)

Call:
plm(formula = log(gsp) ~ log(pcap) + log(pc) + log(emp) + unemp,
    data = Produc, effect = "twoways", model = "random",
    random.method = "swar", index = c("state", "year"))

Balanced Panel: n=48, T=17, N=816

Effects:
                    var   std.dev share
idiosyncratic 1.176e-03 3.429e-02 0.145
individual    6.854e-03 8.279e-02 0.843
time          9.681e-05 9.839e-03 0.012
theta  : 0.9001 (id) 0.5506 (time) 0.5487 (total)

Residuals :
    Min.  1st Qu.   Median  3rd Qu.     Max.
-0.12400 -0.02150 -0.00177  0.01970  0.19700

Coefficients :
              Estimate Std. Error t-value  Pr(>|t|)
(Intercept)  2.3634993  0.1389056 17.0151 < 2.2e-16 ***
log(pcap)    0.0178529  0.0233207  0.7655    0.4442
log(pc)      0.2655895  0.0209824 12.6577 < 2.2e-16 ***
log(emp)     0.7448989  0.0241144 30.8902 < 2.2e-16 ***
unemp       -0.0045755  0.0010179 -4.4952 7.962e-06 ***
```

```
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Total Sum of Squares:    14.989
Residual Sum of Squares: 1.0175
R-Squared      :  0.93212
      Adj. R-Squared :  0.9264
F-statistic: 2783.94 on 4 and 811 DF, p-value: < 2.22e-16
```

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