1. CFF Homework, series 1, to be sent till 30th March

All steps should be explained in detail (preferably by reference to the class assertions).

1.1. Prove that WEP $y^2 - (x^3 - 4x^2 - x + 4)$ is
(a) smooth as a polynomial of $\mathbb{R}[x, y]$ and
(b) singular as a polynomial of $\mathbb{F}_5[x, y]$.
Hint: apply Proposition 3.12

5 points

1.2. Find all singularities of WEP
(a) $y^2 + y(2 - 2x) - (x^3 + x^2 + 3x - 1) \in \mathbb{R}[x, y]$
(b) $y^2 + y(2x + 1) - (x^3 + 2x^2 + 2x) \in \mathbb{F}_3[x, y]$
Hint: use the proof of Proposition 3.12, Lemma 3.10 and Lemma 3.1

10 points

1.3. Find at least 3 points $\alpha \in \mathbb{R}^2$ such that $W := (y^2-(x^3-4x^2-x+4)) \subseteq I_\alpha \subseteq \mathbb{R}[x, y]$. For each such α find an irreducible polynomial $g_\alpha \in I_\alpha \setminus W$.
Hint: apply Proposition 4.3 and Lemma 4.1

5 points