QUESTIONS ON (PROOFS OF) KEY RESULTS OF THE CFF COURSE

2. Valuation rings

2.1. Characterize discrete valuation rings using chain conditions on ideals. Describe all (normalized) discrete valuations on the AFF $K(x)$.

Corresponding claims: 2.10, 2.14

2.2. If L is an AFF over K, $P \in \mathbb{P}_{L/K}$, prove that $\mathcal{O}_P$ is a uniquely defined discrete valuation ring and that $\deg P$ is finite.

Corresponding claim: 2.15

3. Weierstrass equation polynomials

3.1. Define singular and smooth points and say how they can be transformed by affine automorphisms. Formulate and prove characterization of singularities of short WEP's.

Corresponding claims: 3.12, 3.10

4. Coordinate rings

4.1. What is an AFF and how does it can be described by an irreducible affine curve? Which elements of the AFF $K(C)$ for a Weierstrass curve are transcendental?

Corresponding claims: 4.7, 4.8, 4.11

5. Places

5.1. Let $w = yg(x, y) + h(x) + y \in K[x, y]$ where $h \in K[x]$, $g \in K[x, y]$, $m := \text{mult}(h) \geq 2$, $\text{mult}(g) \geq 1$ and L be an AFF over K given by $w(\alpha, \beta) = 0$. What is weighted multiplicity μ of an element of L? Formulate and prove the assertion describing places containing α and β and the corresponding discrete valuation.

Corresponding claims: 5.5, 5.3

5.2. Let L is an AFF over K given by $f(\alpha, \beta) = 0$ with a smooth point $(\gamma_1, \gamma_2) \in V_f(K)$. Formulate and prove the assertion describing places containing $\alpha - \gamma_1$ and $\beta - \gamma_2$ and the corresponding valuation of the point $l_1\alpha + l_2\beta + l_0$.

Corresponding claims: 5.8, 5.9

5.3. Formulate and prove the Weak Approximation Theorem.

Corresponding claim: 5.19

5.4. Describe places of degree 1 of a smooth WEP.

Corresponding claims: 5.13, 5.16, 5.23, 8.3(4)

6. Divisors

6.1. Formulate and prove the theorem on the degree of the positive and negative part of a principal divisor.

Corresponding claims: 6.5, 5.21

6.2. Formulate and prove Riemann theorem and explain what is the genus.

Corresponding claim: 6.10

6.3. Define adeles and formulate and prove the Strong Approximation Theorem.

Corresponding claim: 6.14

7. Weil differentials

7.1. Describe the structure of vector spaces of Weil differentials $\Omega_{L/K}(A)$ and $\Omega_{L/K}(A)$ as subspaces of the space L.

Corresponding claim: 7.3

7.2. Formulate and prove the Riemann-Roch Theorem and its main consequence (about relation of dimension of Riemann-Roch spaces and degrees of divisors).

Corresponding claim: 7.5, 7.6

8. The associative law

8.1. Formulate and proof the assertion characterizing elliptic Weierstrass equation polynomials by property of its points.

Corresponding claims: 8.4, 8.3

8.2. Describe the group structure on a smooth curve given by a WEP

Corresponding claims: 8.8, 8.6

9. Projective curves

9.1. Formulate the correspondence of an AFF given by an affine and by a projective curve and describe geometrical (of type P_a) places of degree given by aprojective curve.

Corresponding claims: 9.4, 9.6