VI. DETERMINE THE RANK OF THE FOLLOWING MATRICES (WITH REAL PARAMETERS)

1. $\begin{pmatrix} 1&2&1&3\\ 2&4&5&1\\ 3&6&7&5\\ 4&8&3&7 \end{pmatrix}$ **2.** $\begin{pmatrix} 1&2&2&3&5\\ 6&15&12&25&42\\ 2&5&4&8&14\\ 1&-1&2&-4&-7 \end{pmatrix}$ **3.** $\begin{pmatrix} a&1&1&1\\ 1&a&1&a\\ 1&1&a&a^2 \end{pmatrix}$ **4.** $\begin{pmatrix} 2(a-1)&(3a+1)&a\\ (1-a)&-2&-1\\ a&2a&a \end{pmatrix}$

5. $\begin{pmatrix} 2(a-1)&(3a+1)&a&2a\\ (1-a)&-2&-1&2\\ a&2a&a&a+1 \end{pmatrix}$ **6.** $\begin{pmatrix} 1&-1&0&-3\\ 7&-2&2&-10\\ 7&-1&1&-9\\ 2&0&-2&-4\\ 6&-1&2&-7 \end{pmatrix}$ **7.** $\begin{pmatrix} 1&1&1&1&1\\ a&2&1&2&a\\ 5&6&7&1&3\\ 1&2&a&2&1\\ 1&0&1&0&1 \end{pmatrix}$

COMPUTE INVERSES OF THE FOLLOWING MATRICES

8. $\begin{pmatrix} 1&2&-1\\ 2&3&0\\ 0&-1&1 \end{pmatrix}$ **9.** $\begin{pmatrix} 1&0&-1\\ -1&1&0\\ 2&1&3 \end{pmatrix}$ **10.** $\begin{pmatrix} 1&2&-3&1\\ 2&3&-1&2\\ 7&-1&4&3\\ 1&1&-2&-1 \end{pmatrix}$ **11.** $\begin{pmatrix} 1&2&-1&1\\ 1&0&0&-1\\ 0&1&1&0\\ 1&2&0&0 \end{pmatrix}$

12. Using the results of these problems write down inverses of matrices made:
a) by interchanging first two rows in the matrix from problem 8; b) by multiplying the fourth row of the matrix from problem 10 by the number 11; c) by adding 7-tuple of the third row to the first row in the matrix from problem 9; d) from the matrix from problem 11 in such a way, that to the first row we put triple of the second row and to the second row we put 5-tuple of the first row.

ANSWERS AND HINTS. **1.** 3 **2.** 3 **3.** 3 for $a \neq 1$, 1 for $a = 1$ **4.** 3 for $a \neq 0, -1, 2$, otherwise 2 **5.** 3 for $a \neq -1$, 2 for $a = -1$ **6.** 3 **7.** 4 for $a \neq 1$, 3 for $a = 1$ **8.** $\begin{pmatrix} 3&-1&3\\ -2&1&-2\\ -2&1&-1 \end{pmatrix}$ **9.** $\begin{pmatrix} 1/2&-1/6&1/6\\ 1/2&5/6&1/6\\ -1/2&-1/6&1/6 \end{pmatrix}$ **10.** $\begin{pmatrix} 0&-1/21&5/42&11/42\\ -1/2&23/42&-5/42&5/21\\ -1/2&13/42&-1/42&1/21\\ 1/2&-5/42&1/21&-25/42 \end{pmatrix}$ **11.** $\begin{pmatrix} 2&2&2&-3\\ -1&-1&-1&2\\ 1&1&2&-2\\ 2&1&2&-3 \end{pmatrix}$ **12.** Inverses of these matrices can be made from inverses to the original matrices: a) by interchanging the first two columns; b) by multiplying the fourth column by 1/11; c) by substracting the 7-tuple of the first column from the third column; d) so that, to the first column we put one third of the second column and to the second column we put one fifth of the first column. (All this follows, for example, from the definition of the inverse of a matrix and from the definition of matrix multiplication. Another possibility is a suitable use of the theorem on multiplication and transformation.)