

Additional questions

Describe a subset  $M \subset \mathbf{R}^2$  and a boundary point  $x \in \mathbf{R}^2$  which belongs to  $M$ .

Describe a subset  $M \subset \mathbf{R}^2$  and a boundary point  $x \in \mathbf{R}^2$  which does not belong to  $M$ .

Describe a subset  $M \subset \mathbf{R}^2$  which is neither open nor closed.

Describe a nonempty subset  $M \subset \mathbf{R}^2$  which has no interior points.

Describe a subset  $M \subset \mathbf{R}^2$  which is closed but not bounded.

Describe a subset  $M \subset \mathbf{R}^2$  which is bounded but not closed.

Describe a subset  $M \subset \mathbf{R}^2$  which is neither closed nor bounded.

Describe a function defined on  $\mathbf{R}^2$  which has both partial derivatives at  $[0, 0]$  but is not continuous at  $[0, 0]$ .

Describe a function defined on  $\mathbf{R}^2$  which is continuous on  $\mathbf{R}^2$  but has neither of the partial derivatives at  $[0, 0]$ .

Describe a function  $f$  defined on  $\mathbf{R}^2$  which is continuous on  $\mathbf{R}^2$  such that  $\frac{\partial f}{\partial x}(0, 0) = 0$  and  $\frac{\partial f}{\partial y}f(0, 0)$  does not exist.

Describe a function  $f$  defined on  $\mathbf{R}^2$  which is continuous on  $\mathbf{R}^2$  such that  $\frac{\partial f}{\partial y}f(0, 0) = 5$  and  $\frac{\partial f}{\partial x}(0, 0)$  does not exist.

Describe a function defined on  $\mathbf{R}^2$  which has at the point  $[0, 0]$  local maximum but not sharp local maximum.

Describe a function defined on  $\mathbf{R}^2$  which has at the point  $[0, 0]$  sharp local maximum but not maximum on  $\mathbf{R}^2$ .

Describe a function defined on  $\mathbf{R}^2$  which has at the point  $[0, 0]$  both local maximum and local minimum.

Describe a function  $f$  defined on  $\mathbf{R}^2$  and a subset  $M \subset \mathbf{R}^2$  containing  $[0, 0]$  such that  $f$  has at  $[0, 0]$  local maximum with respect to  $M$  but not local maximum (with respect to  $\mathbf{R}^2$ ).

Describe a function  $f$  defined on  $\mathbf{R}^2$  such that  $\frac{\partial f}{\partial x}(0, 0) = \frac{\partial f}{\partial y}(0, 0) = 0$  but  $f$  has no local extremum at  $[0, 0]$ .

Find 2-by-2 matrices  $\mathbb{A}$  and  $\mathbb{B}$  such that  $\mathbb{A} \cdot \mathbb{B} \neq \mathbb{B} \cdot \mathbb{A}$ .

Find 3-by-3 matrices  $\mathbb{A}$  and  $\mathbb{B}$  such that  $\mathbb{A} = \mathbb{A}^T$  and  $\mathbb{B} \neq \mathbb{B}^T$ .

Find vectors  $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3 \in \mathbf{R}^3$  which are linearly dependent such that  $\mathbf{v}_1$  and  $\mathbf{v}_2$  are linearly independent.

Find a 4-by-3 matrix of rank 2.

Find a 4-by-3 matrix of rank 3.

Find a 3-by-3 matrix with determinant equal to 17.

Find a linear system of 3 equations with 3 unknowns which has no solution.

Find a linear system of 3 equations with 3 unknowns whose set of solutions is  $\{[1 + t, t, t] : t \in \mathbf{R}\}$ .

Find a linear system of 4 equations with 3 unknowns which has a unique solution.

Find a continuous function  $f$  on  $(0, +\infty)$  such that  $\int_0^{+\infty} f(x)dx = 1$ .

Find a continuous function  $f$  on  $(0, +\infty)$  such that  $\int_0^{+\infty} f(x)dx = -\infty$ .

Find a continuous function  $f$  on  $(0, +\infty)$  such that  $\int_0^{+\infty} f(x)dx$  does not exist.

Find a series which is convergent but not absolutely convergent.

Find a sequence  $\{a_n\}$  such that  $a_n \rightarrow 0$  but the series  $\sum_{n=1}^{\infty} a_n$  diverges.

Find a series which has no sum.