13 Group actions

13.1. Calculate, how many different bracelets can be made with six red and three white beads, using all nine beads.

Solution: 7

13.2. Suppose you have 8 red and 8 blue equilateral triangles. Count the number of ways one can build an equilateral triangle with edges of quadruple sizes

(a) up to rotations,

(b) up to rotations and reflections.

Solutions: (a) 4290 (b) 2220

13.3. Consider the action of the group $G = \mathbf{S}_n$ on the set $\{(a, b) : 1 \leq a, b \leq n\}$, with the permutation π acting on the components, i.e. $\pi((a, b)) = (\pi(a), \pi(b))$. Determine

(a) $|X/\sim|$, the number of cosets of the equivalence $\sim$,

(b) the number of elements of $[(1,1)]_\sim$ and $[(1,2)]_\sim$,

(c) indexes $[G : G_{(1,1)}]$ and $[G : G_{(1,2)}]$.

Solutions: (a) 2*, (b)* $|[(1,1)]_\sim| = n$ *and* $|[(1,2)]_\sim| = n(n-1)$*, (c)* n*,* $n(n-1)$*.*

13.4. Determine how many ways the faces of the regular tetrahedron can be coloured by n colours up to rotations.

Solution: $\frac{1}{12}(n^4 + 11n^2)$.

13.5.* Determine how many ways the faces of the cube can be coloured by n colours up to rotations.

Solution: $\frac{1}{24}(n^6 + 3n^4 + 12n^3 + 8n^2)$.